VISIT…

SECTION 1

GEARS & GEARING

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Design of Bevel Gears

Yesterday's rule of thumb isn't good enough today. With this systematic approach you can quickly predict gear life for a given load capacity.

Wells Coleman

TIME was when the gear designer could rely on rule of thumb, conservative factors of safety, backglances at previous designs. Today he must often design for specific load capacity and life.

Fortunately, though design goals are higher, the approach can be simpler. With the charts given here you can go directly to the proper range of gear sizes; with the rating formulas you can pinpoint the best gear rapidly. The data are based on two key factors, surface durability (pitting resistance) and strength (resistance to tooth breakage). Included are recommendations for diametral pitches, number of teeth, face widths, spiral angles, tooth proportions, mounting design, and gear lubrication—and a completely worked-out design problem. A previous article (see Editor's Note, p 80) compares in detail the various types of bevel gears.

Loads and conditions

You will need to know something about anticipated loads and operating conditions:

Normal operation: What is the normal load and speed, desired number of hours of life? Is operating temperature range to be above the normal 160 to 180 F? If so, you must allow for this in your design.

Peak operation: What will be the maximum torque, the expected duration of maximum torque during gear life, the temperature at peak load?

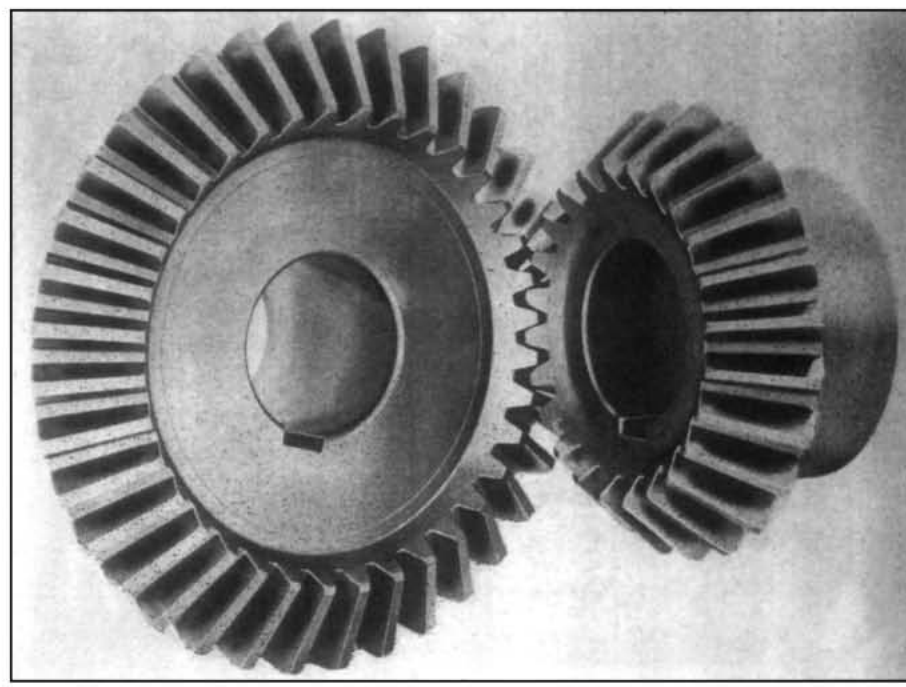
Starting loads: What is the peak starting torque, frequency of occurrence, and duration of starting loads at each start?

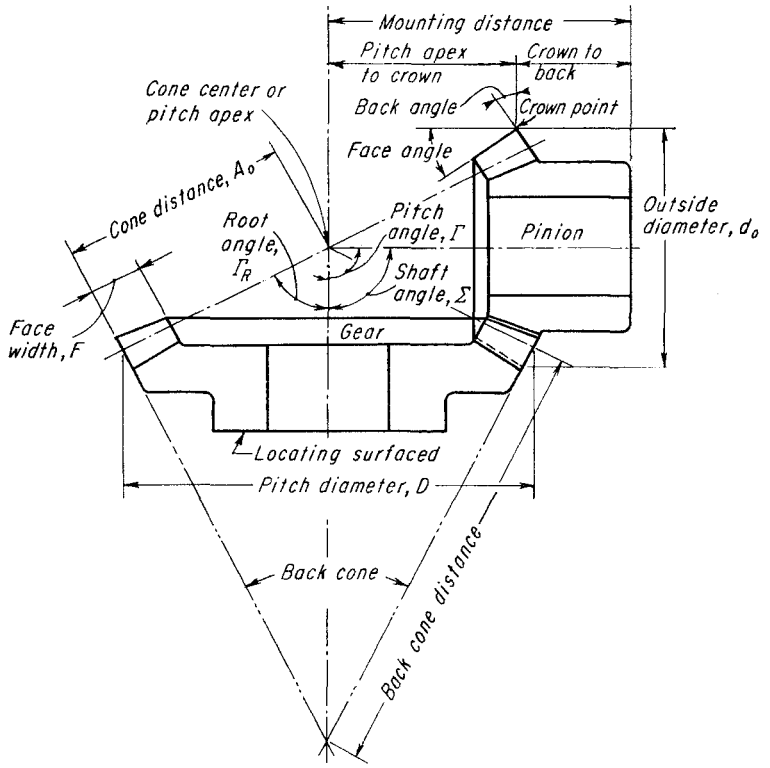
Shock loads: Suggested overload factors are shown in Table I. Shock loads, however, cannot be predicted accurately. Energy absorption methods of load measurement are unreliable because the time duration is so short. Strain-gage measurements must be made with extreme care if results are to be reliable. Repetitive shock is, of course, more damaging than occasional shock loads, but these should not be ignored.

Duration of loads: This information may be known from past experience. More often it is a matter of making an estimate based on a rational premise. Prepare time-torque curves if possible.

Gear lubrication: The rating formulas given in this article assume that the gears will be properly lubricated. Some lubrication hints, however, are also given.

Once the loads and operating conditions are known, the next step is to determine approximate gear size, num-





Symbols

a_o = addendum, in.
A_o = outer cone distance, in.
b_o = dedendum, in.
c = clearance, in.
C_M = material factor
d = pinion pitch dia, in.
d_o = outside dia, in.
D = gear pitch dia, in.
F = face width, in.
h_k = working depth, in.
h_t = whole depth, in.
K = circular thickness factor
I = durability geometry factor
J = strength geometry factor
m = speed ratio
m_F = face contact ratio
n = pinion speed, rpm
N_C = number of teeth in crown gear
N_G = number of gear teeth
N_P = number of pinion teeth
P = maximum operating horsepower, hp
P_d = diametral pitch
t = circular tooth thickness, pinion, in.
T = design pinion torque, lb-in.; also circular tooth thickness, pinion, in.
T' = maximum operating torque, or one-half peak pinion torque, or full peak pinion torque, lb-in.
V = pitch line velocity, ft/min
X_o = pitch apex to crown, in.
δ = dedendum angle
γ = pinion pitch angle, deg
Γ = gear pitch angle, deg
γ_o = pinion face angle
Γ_o = gear face angle
γ_R = pinion root angle
Γ_R = gear root angle
ϕ = pressure angle, deg
Ψ = spiral angle, deg
Σ = shaft angle, deg

bers of teeth, diametral pitch, and face width.

GEAR SIZE

Peak loads

First determine what fraction of the peak load to employ for estimating the gear size. This has been our experience:

If the total duration of the peak load exceeds ten million cycles during the total expected life of the gears, use the peak load for estimating the gear size.

If, however, the total duration of the peak load is less than ten million cycles, use one half the peak load, or the value of the highest sustained load, whichever is greater.

The pinion torque requirement (torque rating) can now be obtained as follows:

$$T = T' \qquad (1)$$

or

$$T = \frac{63{,}000\,P}{n} \qquad (2)$$

where T = design pinion torque, lb-in.
T' = maximum operating pinion torque, or one half peak pinion torque, or full peak pinion torque, as outlined above.
P = maximum operating horsepower
n = pinion speed, rpm

For general industrial gearing the preliminary gear size is based on surface durability (long gear life in preference to minimum weight). The design chart, Fig 1, is from durability tests conducted with right-angle spiral-bevel gears of case-hardened steel. Given pinion torque and the desired gear ratio, the chart gives pinion pitch diameter.

For other materials, multiply the pinion diameter given in Fig 1 by the material factor given in Table II.

Straight bevels and Zerol bevels will be somewhat larger. Multiply the values of pinion pitch diameter from Fig 1 by 1.3 for Zerol bevels and by 1.2 for Coniflex straight bevels. (Zerol and Coniflex are registered trademarks of the Gleason Works.)

For high-capacity spiral bevels (case-hardened, with ground teeth), the preliminary gear size is based on both surface capacity and bending strength. Based on surface capacity, the pinion diameter from Fig 1 should be multiplied by 0.80. Based on bending strength, the pinion diameter is given by Fig 2. Choose the larger pinion diameter of these two.

Statically loaded gears should be

designed for bending strength rather than surface durability. For statically loaded gears which are subject to vibration, multiply the pinion diameter from Fig 2 by 0.70. For statically loaded gears not subject to vibration, multiply the pinion diameter from Fig 2 by 0.60.

Tooth numbers

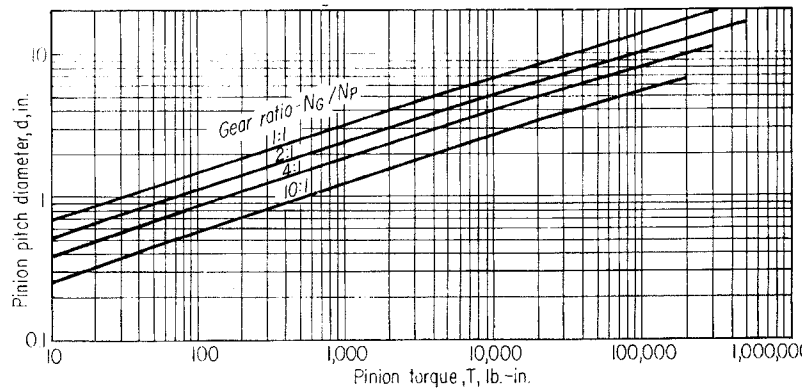
Although tooth numbers are frequently selected in an arbitrary manner, it has been our experience that for most applications the tooth numbers for the pinion from the charts, Fig 3 and 4, will give good results. Fig 3 is for spiral bevels and Fig 4 for straight and Zerol bevels. The number of teeth in the mating gear is of course governed by the gear ratio.

For lapped gears: Avoid a common factor in the numbers of teeth in the gear and mating pinion. This permits better and more uniform wear in the lapping process on hardened gears.

For precision gears: Accuracy of motion is of prime importance; hence the teeth of both pinion and gear should be hardened and ground. Also, use even ratios. Gears made for even ratios are easier to test, inspect, and assemble accurately.

Automotive gears: These are generally designed with fewer pinion

1.. PITCH DIAMETERS BASED ON SURFACE DURABILITY



2.. PITCH DIAMETERS BASED ON TOOTH BENDING STRENGTH

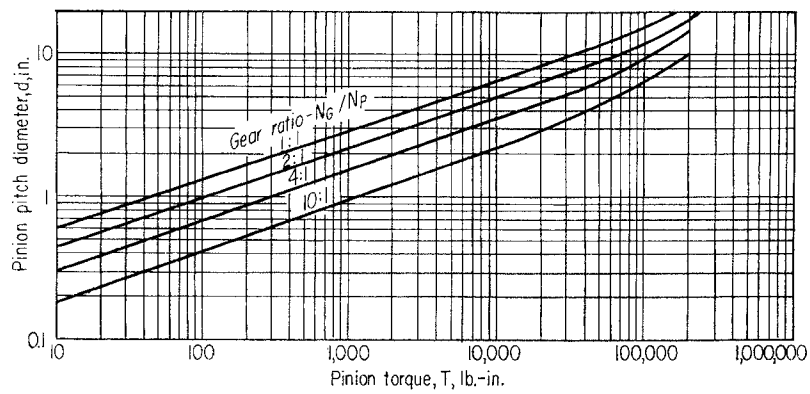


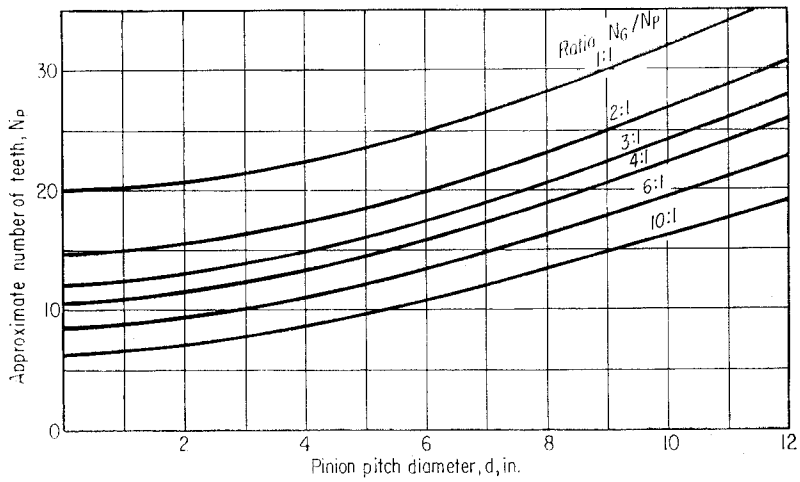
Table I .. Overload factors

Values in this table are for speed decreasing drives; for speed increasing drives add $0.01(N_G/N_P)^2$ to these factors.	POWER SOURCE	CHARACTER OF LOAD ON DRIVEN MACHINE		
		Uniform	**Medium shock**	**Heavy shock**
	Uniform	1.00	1.25	1.75
	Light shock	1.25	1.50	2.00
	Medium shock	1.50	1.75	2.25

Table II .. Material factors for gear mesh

GEAR		PINION		**Material Factor** C_M
	Minimum hardness		**Minimum hardness**	
Case-hardened steel	58 R_C	Case-hardened steel	60 R_C	0.85
Case-hardened steel	55 R_C	Case-hardened steel	55 R_C	1.00
Flame-hardened steel	50 R_C	Case-hardened steel	55 R_C	1.05
Flame-hardened steel	50 R_C	Flame-hardened steel	50 R_C	1.05
Oil-hardened steel	375-425 Brinell	Oil-hardened steel	375-425 Brinell	1.20
Heat-treated steel	250-300 Brinell	Case-hardened steel	55 R_C	1.45
Heat-treated steel	210-245 Brinell	Heat-treated steel	245-280 Brinell	1.65
Cast iron	.....	Case-hardened steel	55 R_C	1.95
Cast iron	.....	Flame-hardened steel	50 R_C	2.00
Cast iron	.....	Annealed steel	160-200 Brinell	2.10
Cast iron	.....	Cast iron	.....	3.10

3 . . NUMBER OF TEETH FOR SPIRAL BEVEL GEARS



4 . . NUMBER OF TEETH FOR STRAIGHT AND ZEROL BEVELS

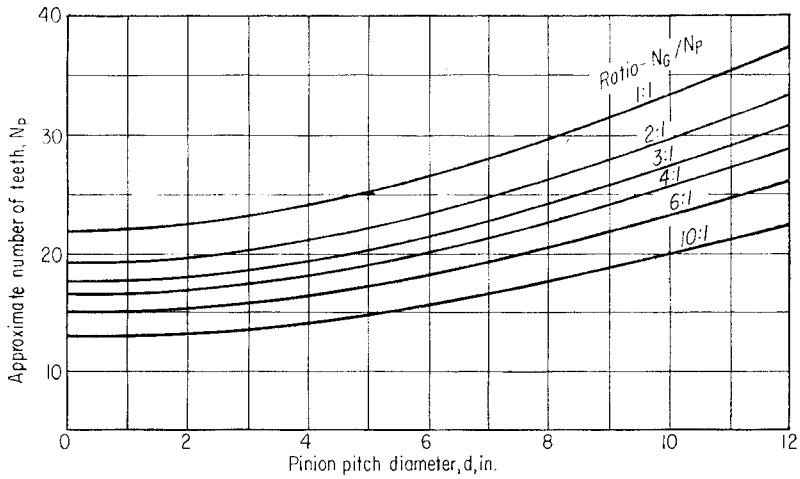


Table III..Pinion teeth for automotive applications

Approximate ratio, N_G/N_P	Preferred number of pinion teeth, N_P	Allowable range, N_P
2.0	17	15-19
2.5	15	12-16
3.0	11	10-14
3.5	10	9-12
4.0	9	8-10
4.5	8	7-9
5.0	7	6-9
6.0	6	5-8
7.0	6	5-7
8.0	5	5-6

teeth. Table III gives suggested tooth numbers for automotive spiral bevel drives. The numbers of teeth in the gear and mating pinion should not contain a common factor.

Face widths

The face width should not exceed 30% of the cone distance for straight-bevel and spiral-bevel gears and should not exceed 25% of the cone distance for Zerol bevel gears. In addition, it is recommended that the face width, F, be limited to

$$F \leqq 10/P_d$$

where P_d is the diametral pitch. Practical values of diametral pitches range from 1 to 64.

The design chart in Fig 5 will give the approximate face width for straight-bevel and spiral-bevel gears. For Zerol bevels the face width given by this chart should be multiplied by 0.83. The chart is based on face width equal to 30% of cone distance.

Diametral pitch

The diametral pitch can now be determined by dividing the number of teeth in the pinion by the pinion pitch diameter. Thus

$$P_d = N_P/d$$

Because tooling for bevel gears is not standardized according to pitch, it is not necessary that the diametral pitch be an integer.

Spiral angle

The spiral angle of spiral-bevel gears should be so selected as to give a face-contact ratio, m_F, of at least 1.25. We have found that for smoothness and quietness, a face-contact ratio of 2.00 or higher will give best results.

The design chart, Fig 6, gives the spiral angle for various face-contact ratios. It is assumed that you have already determined the diametral pitch and face width to obtain the product, P_dF. The curves are based on the equation

$$m_F = P_dF[K_1 \tan \psi - K_2 \tan^3 \psi]$$

where
Ψ = spiral angle
$K_1 = 0.2865$
$K_2 = 0.0171$.

The values for K_1 and K_2 are dependent upon the ratio of face width to outer cone distance of $F/A_0 = 0.3$.

Whenever possible, select the hand of spiral to give an axial thrust that tends to move both the gear and pinion out of mesh. As a second choice, select the hand of spiral to give an

axial thrust that tends to move the pinion out of mesh.

Standard bevel systems

There are three standardized AGMA systems of tooth proportions for bevel gears: 20-deg straight bevel, spiral bevel, and Zerol bevel. There are also several special bevel-gear tooth forms which result in minor modifications to the above proportions. These special forms are used for manufacturing economy or to accommodate special mounting considerations. Because they are very closely tied to the method used in producing the gears, the means of achieving them and the effects they have on standard tooth proportions are beyond the scope of this article.

20-deg straight bevels

General proportions for this system are given in Table IV. The tooth form is based on a symmetrical rack, except where the ratio of tooth top lands on pinion and gear would exceed a 1.5 to 1 ratio. A different value of addendum is employed for each ratio to avoid undercut and to achieve approximately equal strength. If these gears are cut on modern bevel-gear generators they will have a localized tooth bearing. Coniflex gears have this tooth form. To provide uniform clearance, the face cone elements of the gear and pinion blanks are made parallel to the root cone elements of the mating member. This permits the use of larger edge radii on the generating tools, with consequent greater fatigue strength.

Note that the data in Table IV apply only to straight bevel gears that meet the following requirements:

1) The standard pressure angle is 20 deg. See Table V for ratios which may be cut with 14½, 22½, and 25-deg pressure angles.

2) The teeth are full depth. Stub teeth are avoided because of resulting reduction in contact ratio, which can increase both wear and noise.

3) Teeth with long and short addenda are used throughout the system (except on 1:1 ratios) to avoid undercut, increase strength, and reduce wear.

4) The face width is limited to one third the cone distance. The use of a greater face width results in an excessively small tooth size at the inner end of the teeth and, therefore, impractical cutting tools.

The American Gear Manufacturers Assn standard for this system is AGMA 208.02.

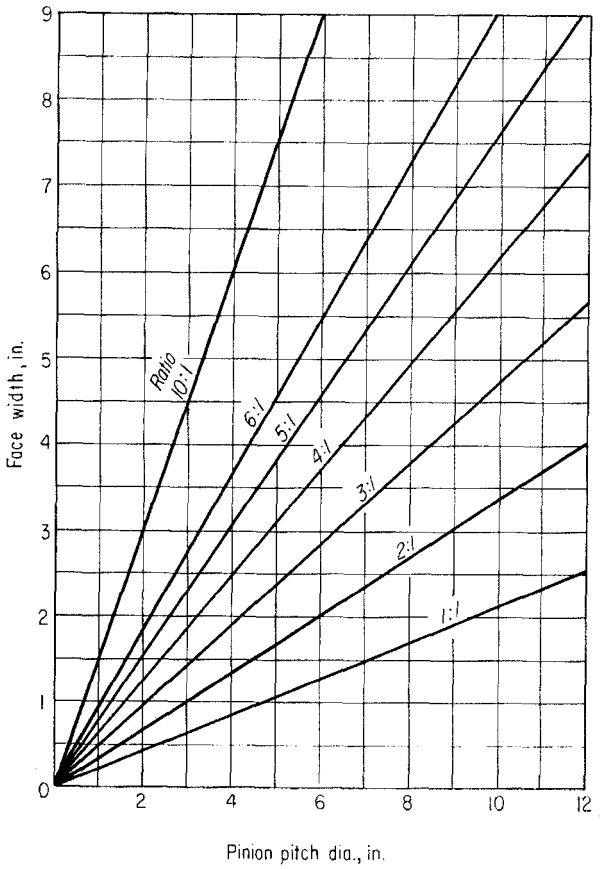
Spiral bevels

Tooth thicknesses (see Table IV) are proportioned so that the stresses in the gear and pinion will be approximately equal with a left-hand pinion driving clockwise or a right-hand pinion driving counterclockwise. These proportions will apply to all gears operating below their fatigue endurance limit. For gears operating above the endurance limit, special thickness proportions will be required. The standard for this system is AGMA 209.02.

The tooth proportions shown are based on the 35-deg spiral angle. A smaller spiral angle may result in undercut and a reduction in contact ratio. The data in this system do not apply to the following:

1) Automotive rear-axle drive gears, which normally are designed with

5 . . FACE WIDTHS FOR SPIRAL BEVELS AT 90° SHAFT ANGLE. FOR ZEROL, MULTIPLY BY 0.83.



6 . . FACE-CONTACT RATIO FOR SPIRAL BEVELS

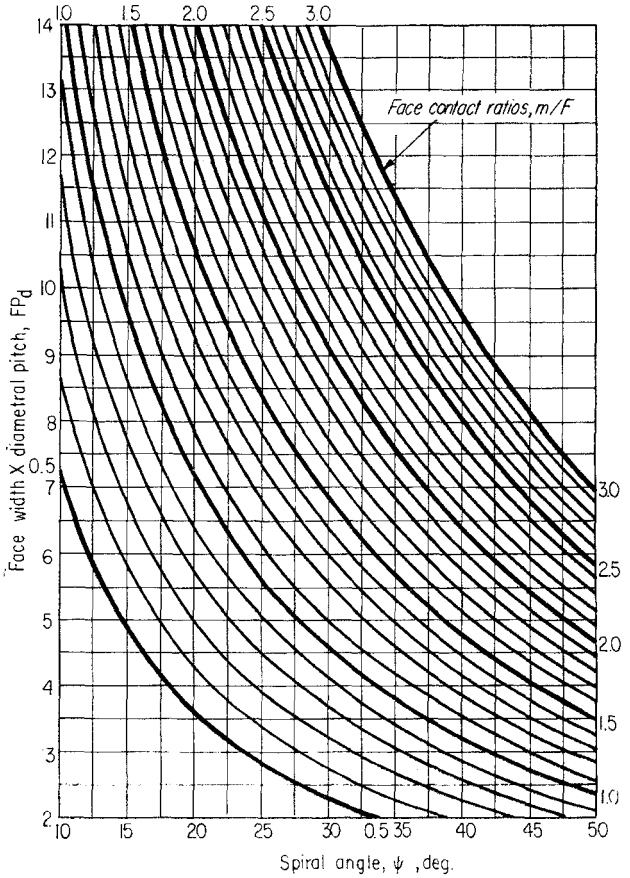


Table IV..Tooth Proportions for Standard Bevel Gears

Item	STANDARD BEVELS (*see Table V for other cases*)		
	Straight	**Spiral**	**Zerol**
Pressure angle, ϕ, deg	20	20	20, 22½, 25
Working depth, h_k, in.	$2.000/P_d$	$1.700/P_d$	$2.000/P_d$
Whole depth, h_t, in.	$(2.188/P_d)+0.002$	$1.888/P_d$	$(2.188/P_d)+0.002$
Clearance, c, in.	$(0.188/P_d)+0.002$	$0.188/P_d$	$(0.188/P_d)+0.002$
Gear addendum, a_o, in.	$\frac{0.540}{P_d}+\frac{0.460}{P_d(N_G/N_P)^2}$	$\frac{0.460}{P_d}+\frac{0.390}{P_d(N_G/N_P)^2}$	$\frac{0.540}{P_d}+\frac{0.460}{P_d(N_G/N_P)^2}$
Face Width, F, in. (Use the smaller value from the two formulas)	$F \leqq \frac{A_o}{3}$ or $F \leqq \frac{10}{P_d}$	$F \leqq \frac{3A_o}{10}$ or $F \leqq \frac{10}{P_d}$	$F \leqq \frac{A_o}{4}$ or $F \leqq \frac{10}{P_d}$
Spiral angle, Ψ, deg	None	25 to 35 (See note 1)	0
Minimum number of teeth (Note 2)	13	12	13
Diametral pitch range	no restriction	12 & coarser	3 & finer
AGMA reference number	208.02	209.02	202.02

Notes

1. 35 deg is the standard spiral angle. If smaller spiral angles are used, undercut may occur and the contact ratio may be less.
2. This is the minimum number of teeth in the basic system. See Table V for equivalent number of teeth in the gear member.

fewer pinion teeth than listed in this system.

2) Helixform and Formate (registered Gleason trademarks) pairs, which are cut with a nongenerated tooth form on the gear.

3) Gears and pinions of 12 diametral pitch and finer. Such gears are usually cut with one of the duplex cutting methods and therefore require special proportions.

4) Ratios with fewer teeth than those listed in Table V.

5) Gears and pinion with less than 25-deg spiral angle.

Zerol bevels

Considerations of tooth proportions to avoid undercut and loss of contact ratio as well as to achieve optimum balance of strength are similar to those for the straight-bevel gear system.

The Zerol system is based on tooth proportions (Table IV) in which the root cone elements do not pass through the pitch cone apex. The face cone element of the mating member is made parallel to the root cone element to produce uniform clearance.

The basic pressure angle is 20 deg. Where needed to avoid undercut, 22½-deg or 25-deg pressure angles are also used (see Table V). The face

Table V..Minimum number of teeth

Pressure angle, deg	STRAIGHT		SPIRAL		ZEROL	
	Pinion	Gear	Pinion	Gear	Pinion	Gear
20 (standard)	16	16	17	17	17	17
	15	17	16	18	16	20
	14	20	15	19	15	25
	13	30	14	20		
			13	22		
			12	26		
14½	29	29	28	28	Not Used	
	28	29	27	29		
	27	31	26	30		
	26	35	25	32		
	25	40	24	33		
	24	57	23	36		
			22	40		
			21	42		
			20	50		
			19	70		
16	Not Used		24	24	Not Used	
			23	25		
			22	26		
			21	27		
			20	29		
			19	31		
			18	36		
			17	45		
			16	59		
22½	13	13	14	14	14	14
					13	15
25	12	12	12	12	13	13

width is limited to one quarter of the cone distance because, owing to the duplex taper, the small-end tooth depth decreases rapidly as the face width increases.

The standard for this system is AGMA 202.02.

Gear-dimension formulas

Table VI gives the formulas for bevel-gear blank dimensions. Tooth proportions are based on data from Table IV. A sample design problem later in the article illustrates the use of these formulas.

Rating formulas

Once the initial gear size has been determined from the above charts, the gears are checked for surface durability and strength, using the following two equations. Surface durability is the resistance to pitting and involves the stress at the point of contact, using Hertzian theory. Strength is the resistance to tooth breakage and refers to the calculation of bending stress in the root of the tooth.

Surface durability:

$$T = \frac{FIK_v}{2K_m}\left(\frac{S_c d}{C_p}\right)^2 \qquad (3)$$

Strength:

$$T = \frac{FJK_v}{2K_s K_m}\left(\frac{S_t d}{P_d}\right) \qquad (4)$$

where

T = maximum allowable torque, lb-in. Use the smaller of the two values.

S_c = allowable contact stress. For recommended values see Table VIII.

S_t = allowable bending stress (also from Table VIII)

d = pinion pitch diameter at larger end of tooth, in.

P_d = diametral pitch at large end of tooth

F = face width, in.

C_p = elastic coefficient (see Table IX)

I = geometry factor (durability) from the design curves in Fig 9 and 10. Fig 9 is for spiral bevel gears with 20-deg pressure angle and 35-deg spiral angle, Fig 10 for straight-bevel and Zerol-bevel gears with 20-deg pressure angle.

J = geometry factor (strength) from the design curves in Fig 11 and 12. Fig 11 is for spiral-bevel gears with 20-deg pressure angle and 35-deg spiral angle. Fig 12 is for straight-bevel and Zerol-bevel gears with 20-deg pressure angle.

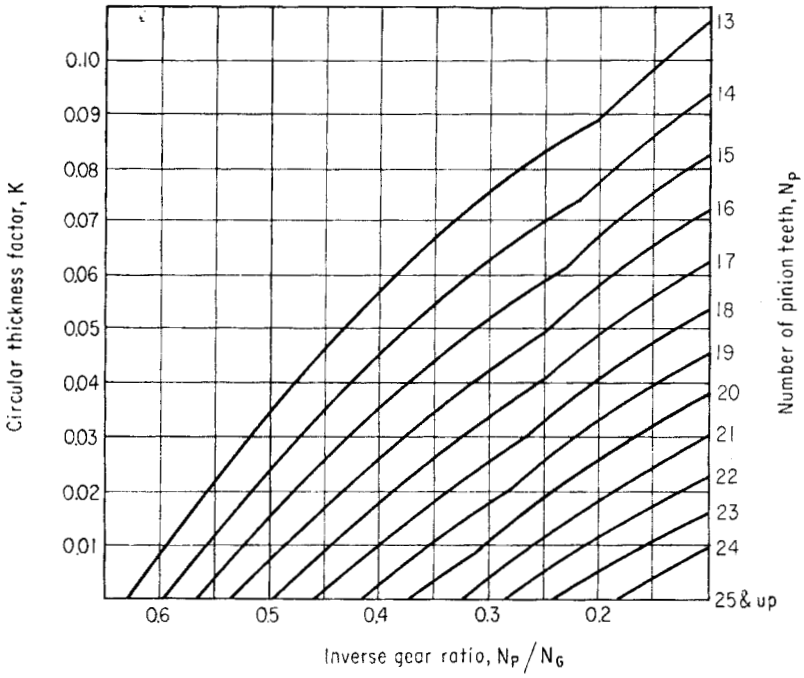
K_m = load distribution factor. Use 1.0 when both gear and pinion are straddle-mounted; use 1.1 when only one member is straddle-mounted. Somewhat higher values may be required if the mountings deflect excessively.

K_v = dynamic factor from the design curves in Fig. 13. Use curve 1 for high-precision ground-tooth gears, curve 2 for industrial spiral bevels, curve 3 for industrial straight-bevel and Zerol-bevel gears.

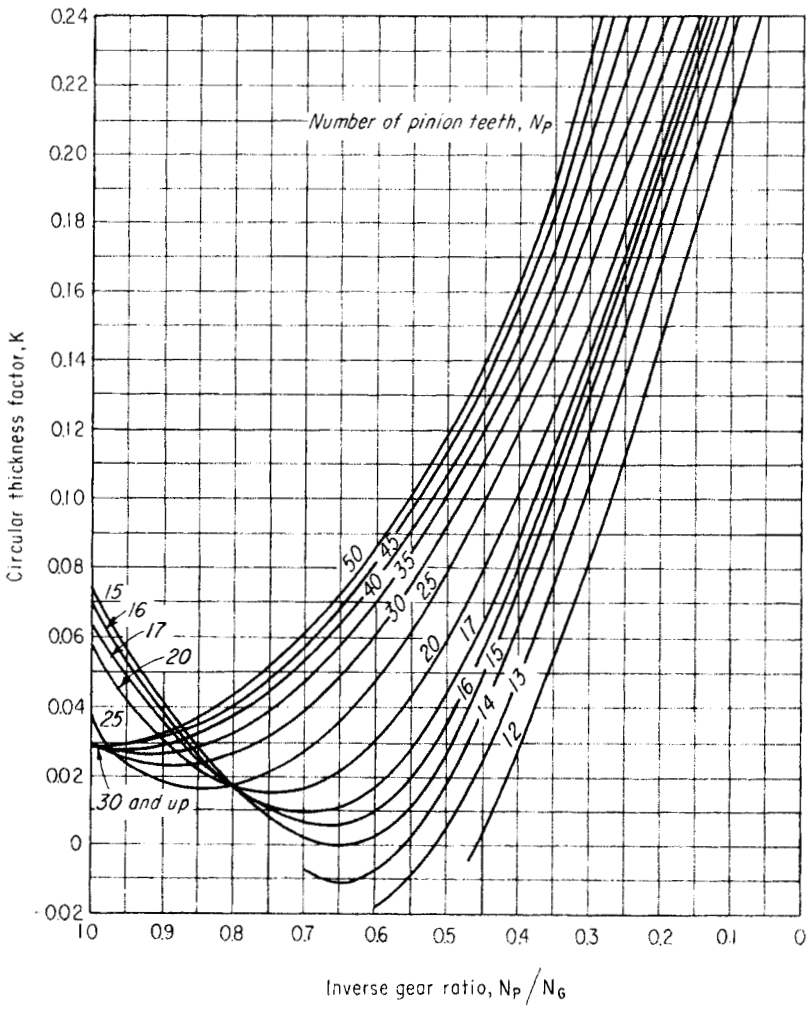
K_s = size factor from Fig 14.

If the computed value of T from either of the above torque equations is less than the design pinion torque,

7 . . CIRCULAR THICKNESS FACTORS, STRAIGHT AND ZEROL BEVELS



8 . . CIRCULAR THICKNESS FACTORS FOR SPIRAL BEVELS



the gear sizes should be increased and another check should be made.

Design example

Select a bevel gear set to connect a small steam turbine to a centrifugal pump with the following specifications: The turbine is to deliver 29 hp at 1800 rpm to a centrifugal pump. The pump is to operate at 575 rpm.

Gear ratio:

$$m = 1800/575 = 3.13$$

Normal operating torque:

$$T' = 63{,}000(29)/1800$$
$$= 1015 \text{ lb-in.}$$

For a centrifugal pump driven by a steam turbine, only light shock with uniform load is anticipated. Therefore an overload factor of 1.25 is selected from Table I.

Design torque:

$$T = 1.25(1{,}015) = 1270 \text{ lb-in.}$$

Because the speed is above 1000 rpm, spiral-bevel gears are used.

Pinion pitch diameter: From Fig 1, for $T = 1{,}270$ lb.-in. and $N_G/N_P = 3.13$, $d = 2.2$ in. Because this is an industrial design, Fig 2 need not be consulted.

Number of teeth: From Fig 3, the pinion will have $N_P = 13$ teeth. Thus, for the gear, $N_G = 13(3.13) = 41$ teeth.

Face width: From Fig 5, the face width of both gears will be approximately $F = 1.1$ in.

Pitch line velocity:

$$V = \frac{3.14(2.2)(1800)}{12}$$
$$= 1030 \text{ ft/min}$$

The approximate size of the gear set has quickly been determined. Now check it for durability and strength using these factors:

$S_c = 200{,}000$ psi, from Table VIII, assuming that both pinion and gear are to be made from case-hardened steel
$C_p = 2800$, from Table IX
$I = 0.116$, from Fig 9
$K_m = 1.1$ for overhung pinion mounting
$K_v = 0.84$ from curve 2, Fig 13

Durability evaluation:

$$T = \frac{(1.1)(0.116)(0.84)(200{,}000)^2(2.2)^2}{(2)(1.1)(2800)^2}$$

$T = 1210$ lb-in.

Since the gears must be designed to carry 1270 lb-in. torque, the gear size should be increased slightly. To approximate the new size, multiply the

Table VI..Bevel-gear dimensions

Dimension *All dimensions in in. and deg*	Formulas, chart or table	
1. Number of pinion teeth, N_P	Figs 3, 4; Tables III, V	
2. Number of gear teeth, N_G	$N_G = m\ N_p$	
3. Diametral pitch, P_d	Practical range, 1 to 64 P_d	
4. Face width, F	Fig 5; Table IV	
5. Working depth, h_k	Table IV	
6. Whole depth, h_t	Table IV	
7. Pressure angle, ϕ	Table IV	
8. Shaft angle, Σ	Practical range, 10 to 180 deg	
	Pinion	**Gear**
9. Pitch diameter; d, D	$d = \frac{N_P}{P_d}$ (Figs 1, 2)	$D = \frac{N_G}{P_d}$
10. Pitch angle; γ, Γ	$\gamma = \tan^{-1}\frac{\sin \Sigma}{\frac{N_G}{N_P} + \cos \Sigma}$	$\Gamma = \Sigma - \gamma$
11. Outer cone distance, A_o	$A_o = \frac{D}{2 \sin \Gamma}$	
12. Circular pitch, p	$p = \frac{3.1416}{P_d}$	
13. Addendum, a_{oP}	$a_{oP} = h_k - a_{oG}$	$a_{oG} =$ (Table IV)
14. Dedendum, b_{oP}	$b_{oP} = h_t - a_{oP}$	$b_{oG} = h_t - a_{oG}$
15. Clearance, c	$c =$ (Table IV)	
16. Dedendum angle, δ (See Note 1)	$\delta_P = \tan^{-1}\frac{b_{oP}}{A_o} + \Delta\delta$	$\delta_G = \tan^{-1}\frac{b_{oG}}{A_o} + \Delta\delta$
17. Face angle of blank; γ_o, Γ_o	$\gamma_o = \gamma + \delta_G$	$\Gamma_o = \Gamma + \delta_P$
18. Root angle; γ_R, Γ_R	$\gamma_R = \gamma - \delta_P$	$\Gamma_R = \Gamma - \delta_G$
19. Outside diameter; d_o, D_o	$d_o = d + 2\ a_{oP} \cos \gamma$	$D_o = D + 2\ a_{oG} \cos \Gamma$
20. Pitch apex to crown; x_o, X_o	$x_o = A_o \cos \gamma - a_{oP} \sin \gamma$	$X_o = A_o \cos \Gamma - a_{oG} \sin \Gamma$
21. Circular thickness; t, T (See Note 2)	$t = p - T$	$T = \frac{p}{2} - (a_{oP} - a_{oG})\frac{\tan \phi}{\cos \Psi} - \frac{K}{P_d}$

Notes:

1. The change in dedendum angle, $\Delta\delta$, is zero for straight bevel and spiral bevel gears; $\Delta\delta$ is given by Table VII for Zerol bevel gears.
2. Factor K is given by Fig 7 for straight bevel and Zerol bevel gears with 20 deg pressure angle, and by Fig 8 for spiral bevel gears with 20 deg pressure angle and 35 deg spiral angle. For other cases K can be determined by the method outlined in "Strength of Bevel and Hypoid Gears" published by the Gleason Works.

Table VII..Formulas for $\Delta\delta$ — Zerol bevel gears

Pressure Angle, deg	Change in Dedendum Angle, $\Delta\delta$, min	Note:
20	$\Delta\delta = \frac{6668}{N_C} - \frac{300}{F}\sqrt{\frac{1}{N_C P_d (\tan \gamma + \tan \Gamma)}} - \frac{14 P_d}{N_C}$	$N_C = 2 P_d A_o =$ number of teeth in crown gear.
22½	$\Delta\delta = \frac{4868}{N_C} - \frac{300}{F}\sqrt{\frac{1}{N_C P_d (\tan \gamma + \tan \Gamma)}} - \frac{14 P_d}{N_C}$	
25	$\Delta\delta = \frac{3412}{N_C} - \frac{300}{F}\sqrt{\frac{1}{N_C P_d (\tan \gamma + \tan \Gamma)}} - \frac{14 P_d}{N_C}$	

Table VIII..Allowable stresses

Material	Heat treatment	Minimum surface hardness	Contact Stress S_C, psi	Bending Stress S_t, psi
Steel	Carburized (*Case Hardened*)	55 R_C	200,000	30,000
Steel	Flame or Induction hardened (*Unhardened root fillet*)	50 R_C	190,000	13,500
Steel	Hardened and Tempered	300 Brinell	135,000	19,000
Steel	Hardened and Tempered	180 Brinell	95,000	13,500
Steel	Normalized	140 Brinell	65,000	11,000
Cast Iron	As Cast	200 Brinell	65,000	7,000
Cast Iron	As Cast	175 Brinell	50,000	4,600
Cast Iron	As Cast	.....	30,000	2,700

Table IX...Elastic coefficients, C_P

Pinion	GEAR MATERIAL Steel	Cast iron	Aluminum	Aluminum bronze	Tin bronze
Steel, $E = 30 \times 10^6$	2800	2450	2000	2400	2350
Cast iron, $E = 19 \times 10^6$	2450	2250	1900	2200	2150
Aluminum, $E = 10.5 \times 10^6$	2000	1900	1650	1850	1800
Aluminum bronze, $E = 17.5 \times 10^6$	2400	2200	1850	2150	2100
Tin bronze, $E = 16.0 \times 10^6$	2350	2150	1800	2100	2050

trial pinion pitch diameter by the square root of the design torque divided by the allowable torque from the first trial.

New pinion pitch diameter:

$$d = 2.2\sqrt{\frac{1270}{1210}} = 2.25 \text{ in.}$$

New face width, from Fig 5:

$F = 1.125$

All other values in Eq 3 remain the same. Use Eq 3 to again check the allowable torque:

$$T = \frac{(1.125)(0.116)(0.84)(200{,}000)^2}{(2)(1.1)(2800)^2} \times (2.25)^2$$

$T = 1290$ lb-in.

This exceeds the required 1270 lb-in.

Strength evaluation

Now make a check of tooth strength, using these factors in Eq 4:

$J = 0.228$, from Fig 11

$K_s = 0.645$, from Fig 14

$S_t = 30{,}000$ psi, from Table VIII

$$P_d = \frac{N_P}{d} = \frac{13}{2.25} = 5.78$$

All other factors remain the same as for the durability evaluation, hence

$$T = \frac{(1.125)(0.228)(0.84)(30{,}000)}{(2)(0.645)(1.1)(5.78)} \times (2.25)$$

$T = 1770$ lb-in. (allowable for strength)

Safety factors:

$$\frac{1290}{1270} = 1.01 \text{ for surface durability}$$

$$\frac{1770}{1270} = 1.39 \text{ for strength}$$

The selected gears, therefore, are correct for surface durability but are conservative for strength. In, say, aerospace applications, strength would dominate over durability and a smaller (and lighter) pinion and gear set would be selected. However, on heavily loaded gears where special surface treatment is given to increase the surface resistance to wear, actual test experience has shown that fatigue breakage in the root fillet rather than a breakdown of the tooth surface does occur. Thus, in applications such as aircraft and automotive, adequate fatigue strength must be assured.

The detail gear dimensions are now obtained.

Gear diameter:

$$D = \frac{N_G}{P_d} = \frac{41}{5.78} = 7.093 \text{ in.}$$

Spiral angle:

$FP_d = (1.125)(5.78) = 6.5$

The spiral angle is now selected with reference to Fig 6. From the curves the face contact ratio, m_F, will be 1.72 with a 35-deg spiral angle or 2.03 with a 40-deg spiral angle. If maximum smoothness and quietness is required, the 40-deg spiral angle is recommended. However, in this case the 35-deg spiral angle should give adequate smoothness. The lower spiral angle reduces the bearing loads and thereby reduces the cost of the unit.

Working depth, Table IV:

$$h_k = \frac{1.700}{5.78} = 0.294$$

Whole depth, Table IV:

$$h_t = \frac{1.888}{5.78} = 0.327$$

MOUNTING DESIGN

With regard to the mountings, the designer should keep three points in mind:

1) Designing the gear blanks, the shafts, bearings, and gear housings to provide the good rigidity as well as accuracy.

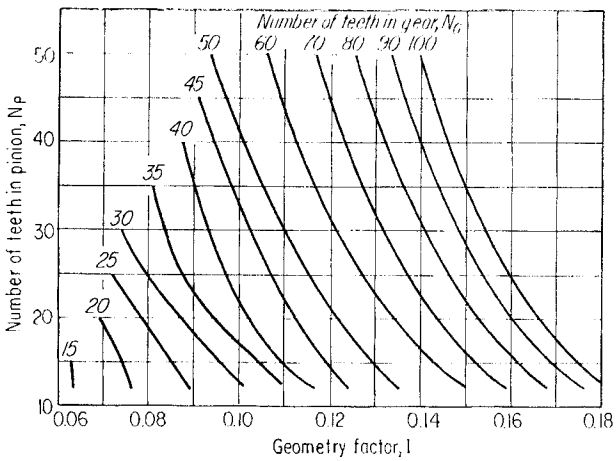
2) Designing the entire unit for ease of assembly.

3) Designing the blanks in a simple geometrical form for ease of manufacture.

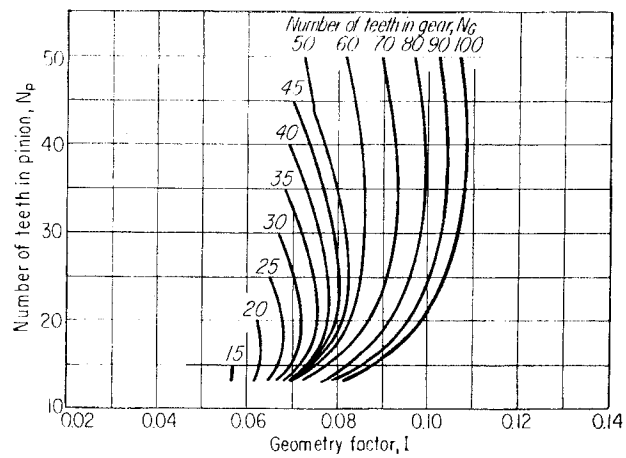
The entire success of the bevel-gear drive depends not only on the design but also on care in manufacturing the unit. The gears must be assembled accurately.

Recommended methods for mounting bevel gears are shown in Fig 15 and 16, and poor vs good design points

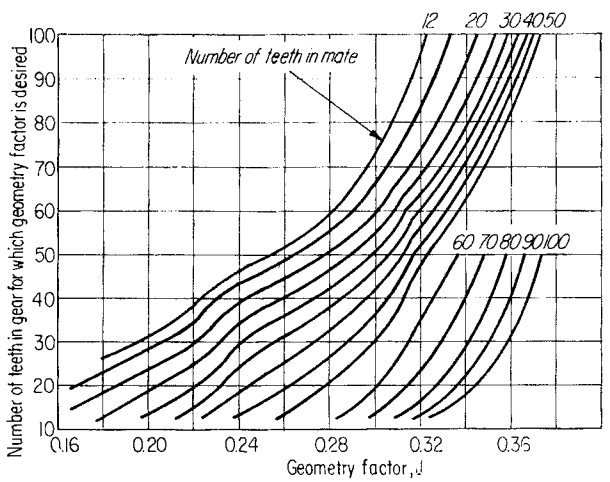
9 . . DURABILITY FACTORS FOR SPIRAL BEVELS



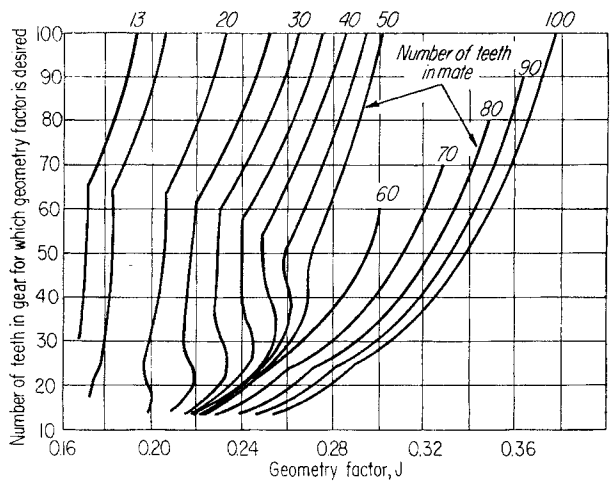
10 . . DURABILITY FACTORS, STRAIGHT AND ZEROL BEVELS



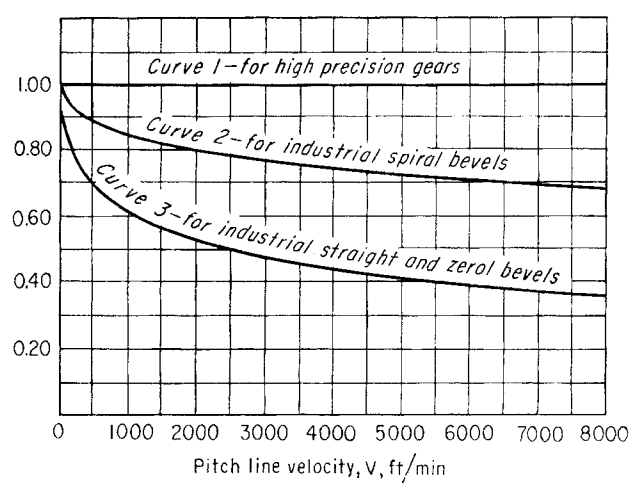
11 . . STRENGTH FACTORS FOR SPIRAL BEVELS



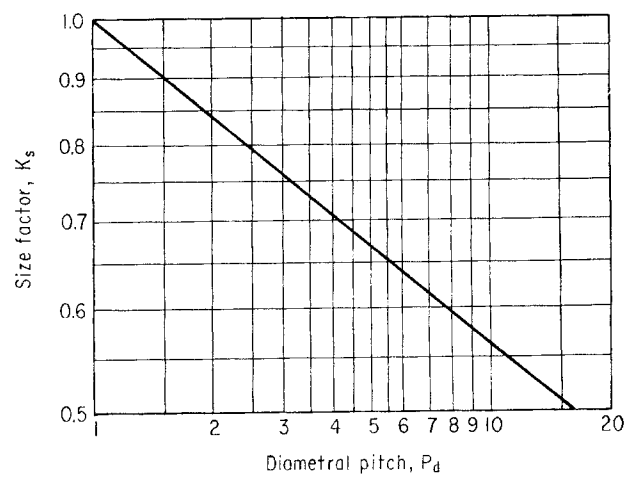
12 . . STRENGTH FACTORS, STRAIGHT AND ZEROL BEVELS



13 . . DYNAMIC FACTORS FOR ALL BEVELS



14 . . SIZE FACTORS FOR ALL BEVELS



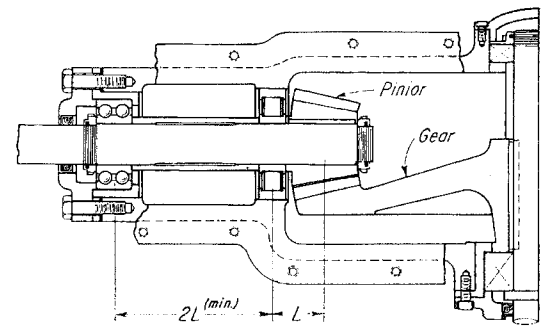
below. As a general rule rolling-friction bearings are superior to plain bearings for bevel gear mountings. This is especially true for spiral-bevel and hypoid gears because these types must be held within recommended limits of deflection and locked against thrust in both directions.

Gear lubrication

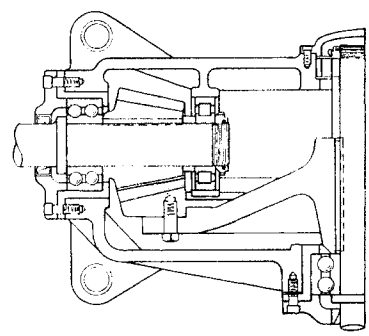
There are two methods recommended for lubricating bevel gears—the splash method and the pressure or jet method. The splash method, in which the gear dips in an oil sump in the bottom of the gear box, is satisfactory for gears operating at peripheral speeds up to 2000 fpm. At higher speeds churning of the oil is likely to cause overheating. For speeds above 2000 fpm a jet of oil should be directed on the leaving side of the mesh point to cover the full length of the teeth on both members. If the drive is reversible, jets should be directed at both the entering and leaving mesh.

Some present-day gear lubricants will operate continuously at temperatures of 200 F and above. However, 160 F is the recommended maximum for normal gear applications. Special oils are not normally required for bevel gears; the lubricants for spur and helical gears are also used for straight, Zerol and spiral bevels.

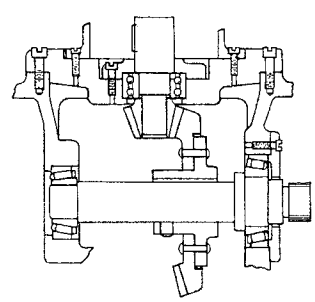
Typical mounting details



15. OVERHUNG MOUNTING: THE DISTANCE BETWEEN BEARING CENTERS SHOULD BE GREATER THAN TWICE THE OVERHUNG DISTANCE, L.

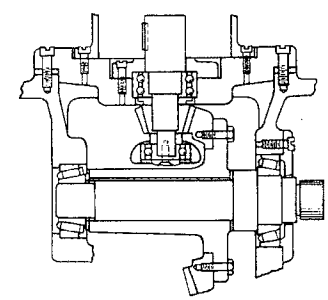


16. STRADDLE-MOUNTING FOR BOTH MEMBERS OF A SPIRAL BEVEL PAIR. FOR HIGH-RIGIDITY APPLICATIONS.



POOR DESIGN

- ☐ Face width too great, more than one third of cone distance.
- ☐ Metal at small end of pinion between teeth and bore too thin.
- ☐ Webbed steel ring gear adds to cost of material and machining.
- ☐ Use of rivets to hold gear to hub introduces danger of runout.
- ☐ A setscrew is inadequate to hold gear in correct axial position and tends to "cock" the gear.
- ☐ Pinion held on shaft only by fit.
- ☐ An overhung pinion cannot be held in line by one double-row bearing.
- ☐ No means of adjustment for gears.



GOOD DESIGN

- ☐ Face width reasonable, less than one third of cone distance.
- ☐ Sufficient metal at small end of pinion to provide strength.
- ☐ Section of ring gear simpler and more direct in design.
- ☐ Screws hold gear to hub.
- ☐ Gear positively held in position.
- ☐ Pinion locked in position by washer and screw.
- ☐ Pinion rigidly supported by addition of inboard bearing.
- ☐ Adjusting washers provided, to be ground to thickness required to correctly position gears.
- ☐ Pinion and bearings can be assembled as a complete unit.

Worksheet Streamlines Bevel-Gear Calculations

B. J. Mumken

The following worksheet neatly gathers together the many mathematical problems that need solving when designing straight bevel-gears. And they are numbered in the correct sequence—no need to hunt "all over the place" as when using formulas in the usual bevel-gear tables. In fact, there are no formulas as such—and, therefore, no need for working with the many Greek symbols found in them.

Instead, the language here is in terms of the actual working operations. For example, space (9) tells you to obtain pitch diameter of the pinion—simply divide the value in space (1) by the value in space (3). And to get root angle for the gear, you are told to subtract the value in space (24) from the value in space (14). Each bracketed number refers you to a value previously filled in.

Just fill in the known values for pinion and gear in the first eight spaces, then work through the sheet, which is based on the Gleason system for 90° straight bevel-gears. Final result (next page) is gear-blank dimensions.

Colored numbers show values obtained in a sample problem worked out by this method.

1	No. of teeth, pinion	40	5	Working depth = $\frac{2.000}{(3)}$	0.200
2	No. of teeth, gear	80	6	Whole depth = $\frac{2.188}{(3)}$ + 0.002 ("D+F")	0.2208
3	Diametral pitch	10	7	Pressure angle	20°
4	Face width	0.750	8	Total backlash	0.003

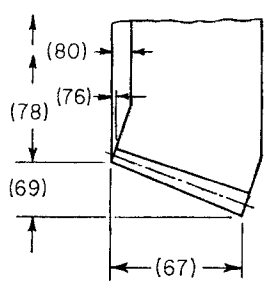
	PINION (Thick underlining indicates working dimensions)			GEAR	
9	Pitch dia. $\frac{(1)}{(3)}$	4.000	10	Pitch dia. $\frac{(2)}{(3)}$	8.000
11	Tan $\frac{(1)}{(2)}$	0.5000	12	Tan $\frac{(2)}{(1)}$	2.0000
13	Pitch angle (11), in deg.	26° 34′	14	Pitch angle (12)	63° 26′
15	2 X cos (13)	1.7888	16	Cone distance $\frac{(10)}{(15)}$	4.4722
18	Addendum (5) − (17)	0.135	17	Addendum = $\frac{\text{(see table)}}{(3)}$	0.065

Gear Addendum for 1 D.P.
Ratio = (No. of gear teeth)/(No. of pinion teeth)

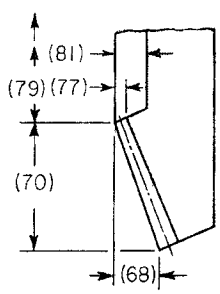
Ratios From	Ratios To	Addendum, in.	Ratios From	Ratios To	Addendum, in.	Ratios From	Ratios To	Addendum, in.	Ratios From	Ratios To	Addendum, in.
1.00	1.00	0.850	1.15	1.17	0.750	1.41	1.44	0.650	1.99	2.10	0.550
1.00	1.02	0.840	1.17	1.19	0.740	1.44	1.48	0.640	2.10	2.23	0.540
1.02	1.03	0.830	1.19	1.21	0.730	1.48	1.52	0.630	2.23	2.38	0.530
1.03	1.05	0.820	1.21	1.23	0.720	1.52	1.57	0.620	2.38	2.58	0.520
1.05	1.06	0.810	1.23	1.26	0.710	1.57	1.63	0.610	2.58	2.82	0.510
1.06	1.08	0.800	1.26	1.28	0.700	1.63	1.68	0.600	2.82	3.17	0.500
1.08	1.09	0.790	1.28	1.31	0.690	1.68	1.75	0.590	3.17	3.67	0.490
1.09	1.11	0.780	1.31	1.34	0.680	1.75	1.82	0.580	3.67	4.56	0.480
1.11	1.13	0.770	1.34	1.37	0.670	1.82	1.90	0.570	4.56	7.00	0.470
1.13	1.15	0.760	1.37	1.41	0.660	1.90	1.99	0.560	7.00	∞	0.460

19	Dedendum = $\frac{2.188}{(3)}$ − (18)	0.0838	20	Dedendum = $\frac{2.188}{(3)}$ − (17)	0.1538
21	Tan $\frac{(19)}{(16)}$	0.0187	22	Tan $\frac{(20)}{(16)}$	0.0343
23	Ded angle (21)	1° 4′	24	Ded angle (22)	1° 58′
25	Face angle (13) + (24)	28° 32′	26	Face angle (14) + (23)	64° 30′
27	Root angle (13) − (23)	25° 30′	28	Root angle (14) − (24)	61° 28′

29	cos (13)	0.8944	30	cos (14)	0.4472
31	[2 x (18)] x (29)	0.2414	32	[2 x (17)] x (30)	0.0581
33	OD = (9) + (31)	4.2415	34	OD = (10) + (32)	8.0581
35	(18) x (30)	0.0603	36	(17) x (29)	0.0581
37	Pitch-apex to crown = [0.5 x (10)] - (35)	3.9396	38	Pitch-apex to crown = [0.5 x (9)] - (36)	1.9419
39	Circular pitch = $\frac{3.1416}{(3)}$	0.3141	41	(18) - (17)	0.0700
40	0.5 x (39)	0.1570	42	(41) x tan (7)	0.0254
44	Circular tooth thickness = (39) - (43)	0.1825	43	Circular tooth thickness = (40) - (42)	0.1316
45	$(44)^3$	0.0060	46	$(44)^3$	0.0022
47	$(9)^2$	16.0000	48	$(10)^2$	64.0000
49	6 x (47)	96.0000	50	6 x (48)	384.000
51	$\frac{(45)}{(49)}$	0.00006	52	$\frac{(46)}{(50)}$	0.0000
53	Chordal tooth thickness = (44) - (51) - [0.5 x (8)]	0.181	54	Chordal tooth thickness = (43) - (52) - [0.5 x (8)]	0.1301
55	$(44)^2$ x (29)	0.0298	56	$(43)^2$ x (30)	0.0077
57	4 x (9)	16.0000	58	4 x (10)	32.0000
59	$\frac{(55)}{(57)}$	0.0019	60	$\frac{(56)}{(58)}$	0.0002
61	Chordal addendum (18) + (59)	0.1369	62	Chordal addendum (17) + (60)	0.0652
63	sin (28)	0.8785	64	sin (27)	0.4771
65	cos (28)	0.4776	66	cos (27)	0.8788



PINION



GEAR

67	(4) x (63)	0.6589	68	(4) x (64)	0.3579
69	(4) x (65)	0.3583	70	(4) x (66)	0.6591
71	$\frac{(16) - (4)}{(16)}$	0.8323			
72	(18) x (71)	0.1124	73	(17) x (71)	0.0541
74	(19) x (71)	0.0697	75	(20) x (71)	0.1280
76	[(72) + (74)] x (30)	0.0815	77	[(73) + (75)] x (29)	0.1629
78	(33) - [2 x (69)]	3.5249	79	(34) - [2 x (70)]	6.7399
80	(76) + mfg. std.	0.125	81	(77) + mfg. std.	0.250

Special Angle Table Simplifies Helical Gear Design

A Helix angle whose cosine is a simple fraction permits rapid calculation of center distances and pitch diameters.

W. U. Matson

HELICAL gears are used when both high speed and high horsepower are required. Although the 45-deg helix angle is most popular for stock gears, as the gear can be used for either parallel or crossed shafts, the large helix angles—30 to 45 deg—impose high thrust loads on bearings when single helicals are used and unless precisely cut and installed increase gear backlash. These helix angles also increase gear weight without proportionally increasing either the strength of the gear or the power and load the helical gear can transmit.

As Table I indicates, based on the use of disks, weights would increase as the square of the diameter. Hence a disk for a 45-deg helix angle of the same normal diametral pitch would be $(1.4142)^2$, or twice as heavy for the same face width as a spur gear (0-deg helix angle). The table also points out the percentage of rise in thrust and bearing loads imposed by the higher-angle gears.

Smaller helix angles—below 30 deg—may increase gear wear slightly but improve backlash tolerance and give lower bearing loads. One helix angle —20° 21′ 50.887″—adds a major advantage, ease of design.

Why a small helix angle

Thrust on the bearings caused by helix angles above 20 deg can be mitigated by double-helical or herringbone teeth. However, face width increases and manufacturing is complicated. Most ball or tapered roller bearings capable of being preloaded can be used with gears of about 20-deg helix angle, as the thrust is less than 50% of the tangential load.

Backlash in the plane of rotation

TABLE I - **Effect of helix angle on various parameters**

HELIX ANGLE ψ DEG	INCREASE IN PITCH DIA OVER STANDARD SPUR GEAR %	AXIAL PRESSURE ANGLE AT PITCHLINE, ϕ	TANGENTIAL PITCHLINE LOAD THRUST %	BEARING %
*0	0	20	0	106.4
10	1.54	20°17′	17.6	106.6
20° 21′ 50.877″	6.66	21°13′	37.1	107.3
30	15.47	22°48′	57.7	108.4
40	30.54	25°25′	83.9	110.7
45	41.42	27°14′	100.0	112.5
	$\%_{inc} = 100\,(1/\cos\psi - 1/\cos 0°)$	$\text{Tan}\,\phi = \frac{\text{Tan}\,\phi_n}{\text{Cos}\,\psi}$	$\%_T = 100 \tan\psi$	$\%_B = 100\,\frac{1}{\text{Cos}\,\phi}$

*Spur gear

TABLE II -Values for one normal diametral

Helix angle, Ψ, = 20° 21′ 50.887″ Pressure angle = 20° For Other Normal Diametral Pitches, Except for

NUMBER OF TEETH IN GEAR, N	PITCH DIAMETER FOR $P_N=1$ D_1	OUTSIDE DIAMETER FOR $P_N=1$ D_{o1}	MEASUREMENT OVER $1.728/P_N$ WIRES FOR $P_N=1$ M_1*
17*	18.1333	20.1333	20.5249
18	19.2000	21.2000	21.5948
19	20.2666	22.2666	22.6641
20	21.3333	23.3333	23.7334
21	22.4000	24.4000	24.8026
22	23.4666	25.4666	25.8714
23	24.5333	26.5333	26.9402
24	25.6000	27.6000	28.0089
25	26.6666	28.6666	29.0773
26	27.7333	29.7333	30.1457
27	28.8000	30.8000	31.2139
28	29.8666	31.8666	32.2821
29	30.9333	32.9333	33.3502
30	32.0000	34.0000	34.4182
31	33.0666	35.0666	35.4860
32	34.1333	36.1333	36.5539
33	35.2000	37.2000	37.6217
34	36.2666	38.2666	38.6894
35	37.3333	39.3333	39.7571
36	38.4000	40.4000	40.8247
37	39.4666	41.4666	41.8923
38	40.5333	42.5333	42.9599
39	41.6000	43.6000	44.0274
40	42.6666	44.6666	45.0949
41	43.7333	45.7333	46.1623
42	44.8000	46.8000	47.2297
43	45.8666	47.8666	48.2971
44	46.9333	48.9333	49.3645
45	48.0000	50.0000	50.4318
46	49.0666	51.0666	51.4990
47	50.1333	52.1333	52.5663
48	51.2000	53.2000	53.6336
49	52.2666	54.2666	54.7008
50	53.3333	55.3333	55.7680
51	54.4000	56.4000	56.8352
52	55.4666	57.4666	57.9022
53	56.5333	58.5333	58.9695
54	57.6000	59.6000	60.0367
55	58.6666	60.6666	61.1037
56	59.7333	61.7333	62.1709
57	60.8000	62.8000	63.2380
58	61.8666	63.8666	64.3050
59	62.9333	64.9333	65.3722
60	64.0000	66.0000	66.4392
61	65.0666	67.0666	67.5062
62	66.1333	68.1333	68.5733
63	67.2000	69.2000	69.6404
64	68.2666	70.2666	70.7073
65	69.3333	71.3333	71.7743
66	70.4000	72.4000	72.8413
67	71.4666	73.4666	73.9081
68	72.5333	74.5333	74.9752
69	73.6000	75.6000	76.0422
70	74.6666	76.6666	77.1091
71	75.7333	77.7333	78.1761
72	76.8000	78.8000	79.2432
73	77.8666	79.8666	80.3100
74	78.9333	80.9333	81.3770
75	80.0000	82.0000	82.4439
76	81.0666	83.0666	83.5107
77	82.1333	84.1333	84.5777
78	83.2000	85.2000	85.6446
79	84.2666	86.2666	86.7115
80	85.3333	87.3333	87.7784
81	86.4000	88.4000	88.8454
82	87.4666	89.4666	89.9121
83	88.5333	90.5333	90.9790
84	89.6000	91.6000	92.0460
85	90.6666	92.6666	93.1128
86	91.7333	93.7333	94.1797
87	92.8000	94.8000	95.2465
88	93.8666	95.8666	96.3133
89	94.9333	96.9333	97.3802
90	96.0000	98.0000	98.4471
91	97.0666	99.0666	99.5139
92	98.1333	100.1333	100.5807
93	99.2000	101.2000	101.6476
94	100.2666	102.2666	102.7144
95	101.3333	103.3333	103.7813
96	102.4000	104.4000	104.8482
97	103.4666	105.4666	105.9150
98	104.5333	106.5333	106.9818
99	105.6000	107.6000	108.0486
100	106.6666	108.6666	109.1154
101	107.7333	109.7333	110.1823
102	108.8000	110.8000	111.2491
103	109.8666	111.8666	112.3158
104	110.9333	112.9333	113.3826
105	112.0000	114.0000	114.4495
106	113.0666	115.0666	115.5163
107	114.1333	116.1333	116.5831
108	115.2000	117.2000	117.6499
109	116.2666	118.2666	118.7166
110	117.3333	119.3333	119.7834
111	118.4000	120.4000	120.8503
112	119.4666	121.4666	121.9170
113	120.5333	122.5333	122.9838
114	121.6000	123.6000	124.0506

**For odd tooth gears—divide measurement over wires by 2 and make radial measurement.*

increases with helix angle. For larger helix angles, greater precision of gear cutting is required where the backlash desired is small in the direction of rotation.

Manufacture by hobbing of gears with high helix angles is sometimes limited by the extent to which the hobs can be rotated or swiveled. Gears of high helix angle sometimes require special setups and equipment.

Face width is usually planned to give complete pitch-line contact overlap. For very small helix angles the face required to do this is large, as sin ψ becomes small in the denominator of the equation. The small faces afforded by large helix angles can not be used because of forces during cutting. But, face widths for gears of about 20-deg helix angle are of reasonable size for the various normal diametral pitches.

Ease of design can not be ignored.

pitch for full depth helical gears

Number of Teeth, Divide Values by Normal Diametral Pitch (Hob Cutter Pitch).

NUMBER OF TEETH IN GEAR, N	PITCH DIAMETER FOR $P_N=1$ D_1	OUTSIDE DIAMETER FOR $P_N=1$ D_{o1}	MEASUREMENT OVER $1.728/P_N$ WIRES FOR $P_N=1$ M_1*
115	122.6666	124.6666	125.1174
116	123.7333	125.7333	126.1842
117	124.8000	126.8000	127.2510
118	125.8666	127.8666	128.3177
119	126.9333	128.9333	129.3845
120	128.0000	130.0000	130.4513
121	129.0666	131.0666	131.5180
122	130.1333	132.1333	132.5848
123	131.2000	133.2000	133.6516
124	132.2666	134.2666	134.7182
125	133.3333	135.3333	135.7850
126	134.4000	136.4000	136.8519
127	135.4666	137.4666	137.9186
128	136.5333	138.5333	138.9854
129	137.6000	139.6000	140.0522
130	138.6666	140.6666	141.1188
131	139.7333	141.7333	142.1856
132	140.8000	142.8000	143.2524
133	141.8666	143.8666	144.3191
134	142.9333	144.9333	145.3859
135	144.0000	146.0000	146.4527
136	145.0666	147.0666	147.5193
137	146.1333	148.1333	148.5861
138	147.2000	149.2000	149.6529
139	148.2666	150.2666	150.7196
140	149.3333	151.3333	151.7864
141	150.4000	152.4000	152.8532
142	151.4666	153.4666	153.9198
143	152.5333	154.5333	154.9866
144	153.6000	155.6000	156.0534
145	154.6666	156.6666	157.1201
146	155.7333	157.7333	158.1869
147	156.8000	158.8000	159.2536
148	157.8666	159.8666	160.3203
149	158.9333	160.9333	161.3870
150	160.0000	162.0000	162.4538
151	161.0666	163.0666	163.5205
152	162.1333	164.1333	164.5872
153	163.2000	165.2000	165.6540
154	164.2666	166.2666	166.7206
155	165.3333	167.3333	167.7874
156	166.4000	168.4000	168.8542
157	167.4666	169.4666	169.9208
158	168.5333	170.5333	170.9876
159	169.6000	171.6000	172.0543
160	170.6666	172.6666	173.1210
161	171.7333	173.7333	174.1878
162	172.8000	174.8000	175.2546
163	173.8666	175.8666	176.3213
164	174.9333	176.9333	177.3881
165	176.0000	178.0000	178.4548
166	177.0666	179.0666	179.5214
167	178.1333	180.1333	180.5882
168	179.2000	181.2000	181.6549
169	180.2666	182.2666	182.7216
170	181.3333	183.3333	183.7883
171	182.4000	184.4000	184.8550
172	183.4666	185.4666	185.9217
173	184.5333	186.5333	186.9884
174	185.6000	187.6000	188.0551
175	186.6666	188.6666	189.1218
176	187.7333	189.7333	190.1885
177	188.8000	190.8000	191.2553
178	189.8666	191.8666	192.3219
179	190.9333	192.9333	193.3887
180	192.0000	194.0000	194.4554
181	193.0666	195.0666	195.5220
182	194.1333	196.1333	196.5887
183	195.2000	197.2000	197.6555
184	196.2666	198.2666	198.7221
185	197.3333	199.3333	199.7889
186	198.4000	200.4000	200.8556
187	199.4666	201.4666	201.9222
188	200.5333	202.5333	202.9890
189	201.6000	203.6000	204.0557
190	202.6666	204.6666	205.1223
191	203.7333	205.7333	206.1891
192	204.8000	206.8000	207.2558
193	205.8666	207.8666	208.3225
194	206.9333	208.9333	209.3892
195	208.0000	210.0000	210.4560
196	209.0666	211.0666	211.5226
197	210.1333	212.1333	212.5894
198	211.2000	213.2000	213.6561
199	212.2666	214.2666	214.7227
200	213.3333	215.3333	215.7894
300	320.0000	322.0000	322.4590
400	426.6666	428.6666	429.1270
500	533.3333	535.3333	535.7946
600	640.0000	642.0000	642.4618
700	746.6666	748.6666	749.1288
800	853.3333	855.3333	855.7958
900	960.0000	962.0000	962.4627
1000	1066.6666	1068.6666	1069.1295

And the cosine of 20° 21′ 50.887″ is equal to 0.9375 = $^{15}/_{16}$, a simple fraction. Thus, many design calculations can be done in longhand or with slide rule, eliminating some tedious calculations. For example, if a 20-deg angle is used the cosine value equals 0.93969262, much harder to manipulate than $^{15}/_{16}$, although the difference in helix angle is slight.

There are many other angles whose cosines are simple fractions such as $^{4}/_{5}$, $^{6}/_{7}$, $^{7}/_{8}$, etc. These range from helix angles of 36° 52′ 11″ to 16° 15′ 36″ and many have simple sine values. Many of these might be good choices.

Why standardize on an angle

Manufacturing and engineering can be simplied, as stocks of helical gears could be made and cataloged for the trade as is done with spur gears. In addition, expensive cam guides for gear shapers could be purchased with certainty of full use by gear manu-

EXAMPLE—DESIGN OF HELICAL GEARS — $\psi = 20°\ 21'\ 50.887''$

$P_N = 48,\quad PA = 20°,\quad N_P = 38,\quad N_G = 64$

STEP	FORMULA	PINION	GEAR
1	$P = (P_N)(15/16)$	45	45
2	PA = pressure angle (given)	20°	20°
3	P_N = normal diametral pitch (given)	48	48
4	N = number of teeth (given)	38	64
5	ψ = helix angle (given)	20° 21′ 50.887″	20° 21′ 50.887″
6	$D = D_1(\text{table})/P_N$	0.8444	1.4222
7	$a = 1/P_N$	0.02083	0.02083
8	Hand	Right	Left
9	Whole depth	Not required	Not required
10	$D_o = D_{01}(\text{table})/P_N$	0.8861	1.4638
11	$G = 1.728/P_N$	0.036	0.036
12	$M = M_1(\text{table})/P_N$	0.8949	1.4730
13	$F = 9.03/P_N = 0.188$	Use 0.250	Use 0.218
14	$C = (D_G + D_o)/2$	1.1333	1.1333

facturers. And, it is now possible to achieve enough precision to manufacture fully interchangeable helical gears rather than furnish them in pairs of special design. Mathematical tables such as those that follow can be used to simplify design and manufacturing calculations.

Tables reduce mathematics

Helical gears can be manufactured by hobbing with standard spur gear cutters as easily as spur gears, once the essential calculations have been made. For this reason, the following set of helical gears with a helix angle of 20° 21′ 50.887″ is suggested for use where smoothness, lack of vibration and quietness of drive are desired.

Tables have been prepared in terms of one normal pitch, with the effects on the dimensions caused by the helix angle already incorporated. To obtain the values to be tabulated for a gear, the one-normal-pitch dimensions are divided by the normal (or cutter) diametral pitch. Steps in using the table are:

1) Complete pitch diameters by either of these two methods:

a) By formula:

$$D = N/P = N/P_N \cos\psi$$
$$= N/P_N(15/16)$$
$$= 16N/15P_N$$

b) By table:

Choose value D_1 and divide by the cutter or normal diametral pitch, the cutter or normal diametral pitch, P_N.

2) Compute outside diameter of gears by either of these two methods:

a) By formula

Add $2/P_N$ to pitch diameter

b) By table:

Choose value D_{01} and divide by normal diametral pitch, for full-depth involute gears only.

3) Compute minimum face width, F. Very wide faces should be avoided.

$$F = \pi/P_N \sin\psi = 9.03/P_N$$

4) Compute measurement over wires. This has been done for one normal diametral pitch; hence divide table M_1 values by normal diametral pitch, P_N. Use radial measurements for odd-toothed gears.

5) Compute thrust. Any ball or tapered roller bearing of the same shaft bore that has provisions for axial preloading or thrust will usually be adequate to handle thrust of these helical gears. Where doubt exists, thrust load W_T equals W tan ψ, which is 0.371 W. W is tangential pitch-line load. Bearings should be free of end play and kept close to gears for rigid support.

How to use tables

A design example is shown above. A summary of the formulas used for the example in order of gear tabulation for drawings would be:

1) Diametral pitch in plane of rotation, P

$$P = P_N \cos\psi$$

2) Normal pressure angle, ϕ_N
3) Normal diametral pitch, P_N
4) Number of teeth, N
5) Helix angle, ψ
6) Pitch diameter, D

$$D = D_1(\text{table})/P_N$$

7) Addendum, a

$$a = 1/P_N$$

8) Hand, one gear must be left, the other right
9) Whole depth, not required
10) Outside diameter, D_o

$$D_o = D_{01}(\text{table})/P_N$$

11) Wire diameter, G

$$G = 1.728/P_N$$

12) Measurement over wires

$$M = M_1(\text{table})/P_N$$

13) Minimum face width, F

$$F = 9.03/P_N$$

14) Center distance, C

$$C = (D_G + D_P)/2$$

All other values such as index gears, feed gears, machine constant, feed, material, etc, must be determined on the basis of gear application and machine type and cutter type to be used. This information must come from the gear consultant and is based on manufacturing equipment available for generating the gear teeth.

Form Cutters for Helical Gears

Oliver Saari

Most handbooks and other sources still give the formula for the selection of the proper spur tooth cutter for milling involute helical gear teeth as being

$$N_c = \frac{N}{\cos^3 \psi} \qquad (1)$$

where

N = actual number of teeth in gear to be cut
ψ = helix angle of gear to be cut
N_c = number of spur gear teeth for which cutter has been made.

No spur-tooth form cutter can give an exact involute shape when used to mill helical teeth—in fact, even for spur gears the form is correct for only one number of teeth in the range specified for a given cutter. But Formula (1) gives a needlessly poor approximation, particularly with small numbers of teeth and helix angles above 20°. Its continued presentation to the gear-cutting public is strange in view of the fact that thirty years ago Ernest Wildhaber published a much more exact and almost as simple formula for finding the cutter number (*AM*—Dec. 20, 1923). This formula is

$$N_c = \frac{N}{\cos^3 \psi} + (P_n D_c) \tan^2 \psi \qquad (2)$$

where the symbols are the same as before, and in addition

P_n = normal diametral pitch of gear and cutter
D_c = pitch diameter of cutter = OD of cutter − 2 × dedendum of gear

This formula gives values which are theoretically exact in both tooth-thickness and pressure angle at one point of the tooth profile, and the spur cutter itself provides a crowning or relieving effect above and below this point.

Possibly one reason that this simple formula has not been included in the readily available

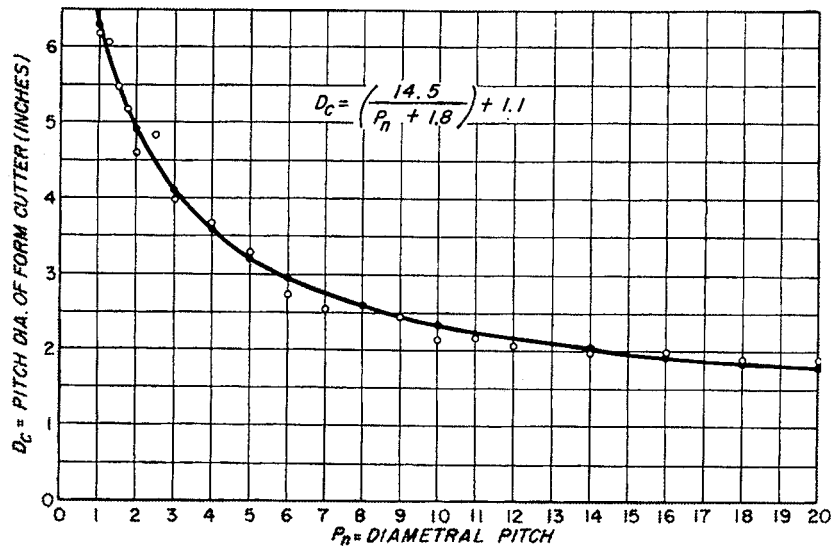

$D_c = \left(\frac{14.5}{P_n + 1.8}\right) + 1.1$
D_c = PITCH DIA. OF FORM CUTTER (INCHES)
P_n = DIAMETRAL PITCH
0 1 2 3 4 5 6
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

FIG. 1 . . . Empirical relationship of gear-cutter pitch diameter to diametral pitch (Standard ITW cutters)

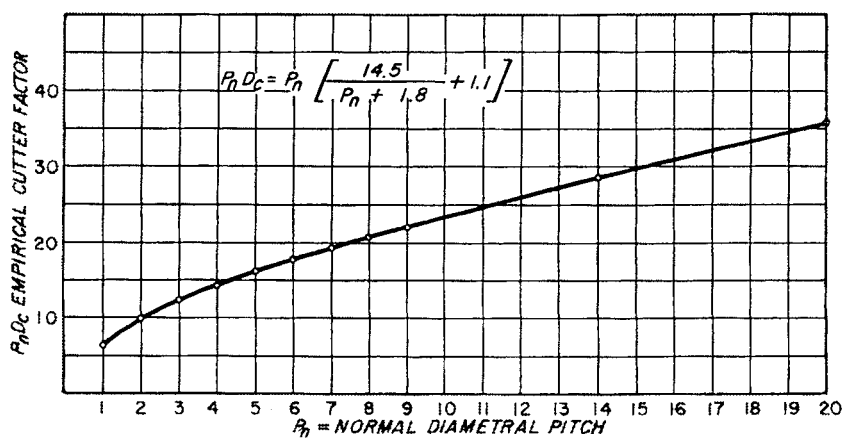

$P_n D_c = P_n \left[\frac{14.5}{P_n + 1.8} + 1.1\right]$
$P_n D_c$ EMPIRICAL CUTTER FACTOR
P_n = NORMAL DIAMETRAL PITCH
10 20 30 40
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20

Fig. 2 . . . Empirical relationship of $P_n D_c$ to diametral pitch for standard ITW cutters

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gear literature is the fact that it involves the cutter pitch diameter. This varies with the diametral pitch and with the various manufacturers. It is not generally known to the tool designer or anyone except the man in the shop who actually gets the cutter from the tool crib. Then it is too late to do anything but "cut and try."

This difficulty can be eliminated by establishing an empirical relationship between the cutter diameter and diametral pitch. Spur cutter sizes can be directly related to diametral pitches, even though such a relation may not be intentional on the part of the manufacturer, because the proportions of the tooth form and the size of hole required almost automatically dictate the "natural" size of the cutters. A curve of this relationship, together with an empirical formula, is shown in Fig. 1. Even though the individual cutters may deviate slightly from this curve, the errors caused by the deviation are far smaller than those resulting if the effects of cutter diameter are ignored altogether.

The combined relationship of the product $P_n D_c$, which appears in the exact equation (2), to the diametral pitch is shown in the curve and empirical formula of Fig 2. This curve and Equation (2) are sufficient for all calculations in the range of pitches from one to 20. A combined formula which may be used alternatively if Curve (2) is not available is

$$N_c = \frac{N}{\cos^3 \psi} + P_n \left(\frac{14.5}{P_n + 1.8} + 1.1 \right) \tan^2 \psi \quad (3)$$

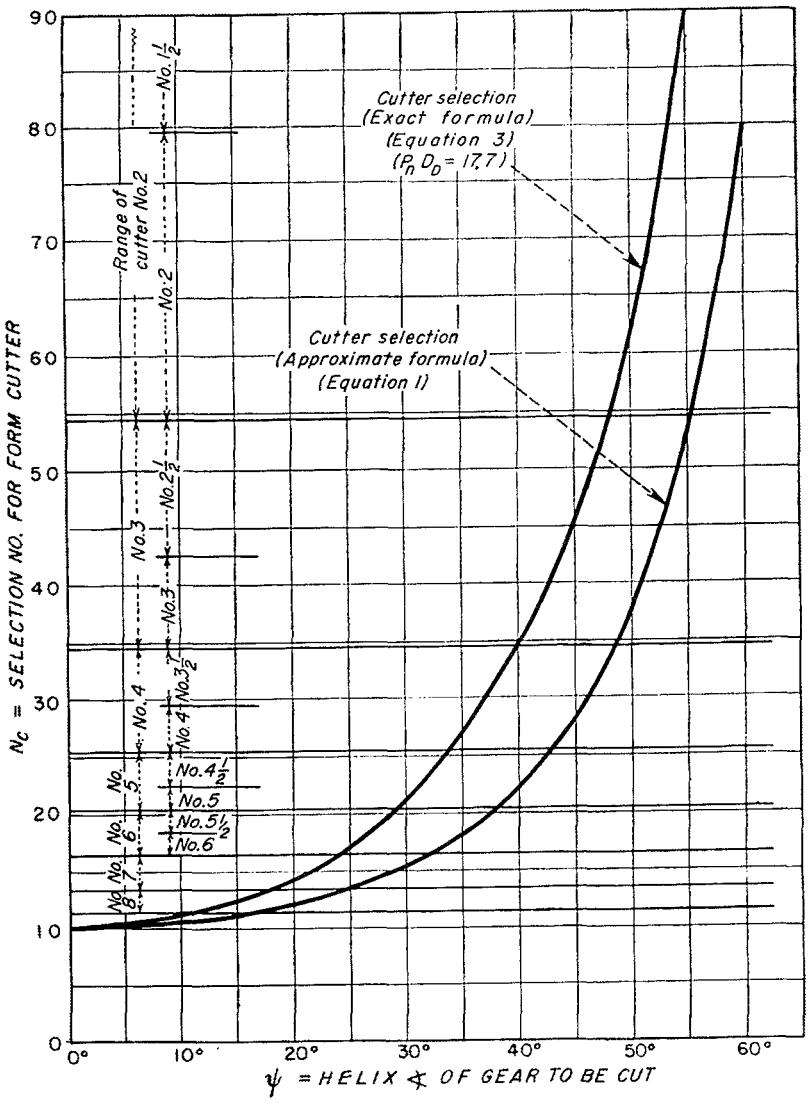


FIG. 3 ... Curves for 6-DP gear, 10 teeth, showing the variation in form-cutter number determined by the exact and approximate selection formulas

A comparison of the values obtained by the use of Equations (1) and (3) for one particular case—10-tooth, 6-pitch gears of various helix angles — is shown by the curves of Fig 3. These curves show that the exact formula gives a better choice of cutter number for all helix angles above 17°.

Planetary Gear Systems

Efficiency and Speed-Ratio Formulas

Here are complete torque, speed, and power analysis, a new method for finding efficiencies and final formulas ready for use.

John H. Glover

THE remarkably high speed reductions that are produced in relatively small spaces by planetary gear systems fascinate gear designers. New gear arrangements are periodically developed—you might say invented—and it is surprising to note how similar many of them look when compared to each other. Yet, slight differences can cause one arrangement to have a much higher or lower speed-reduction ratio than another.

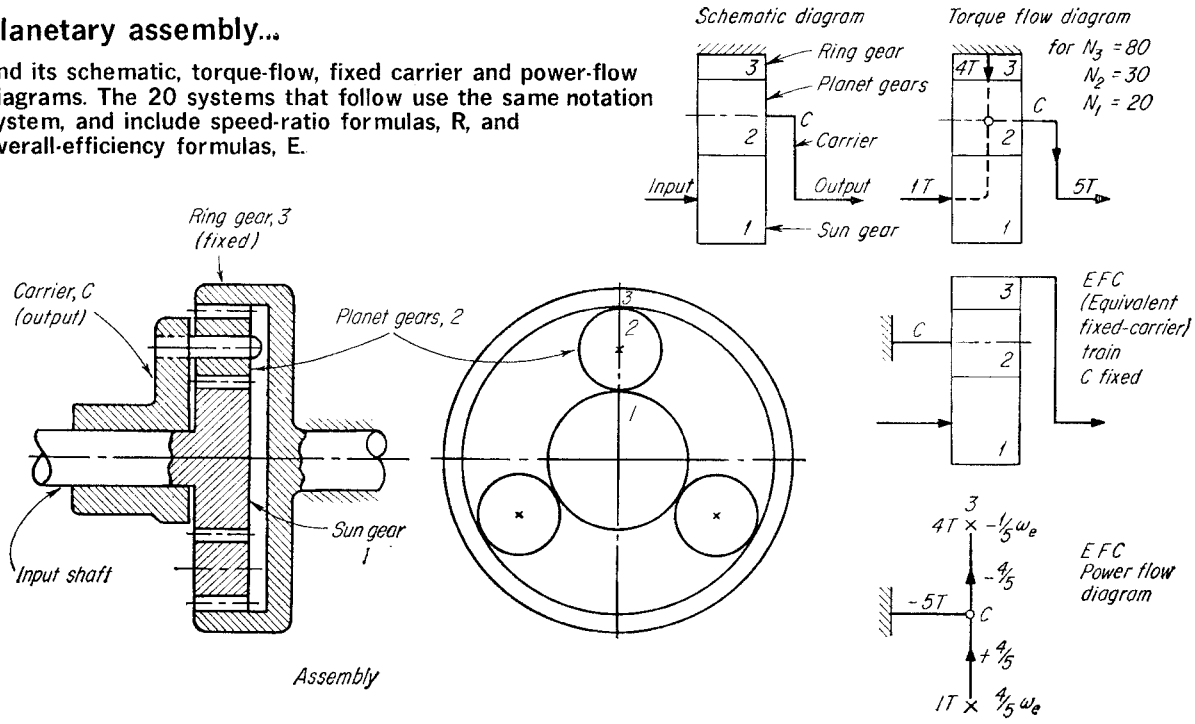
But speed reduction is not the only design criterion. Just as important is the overall efficiency of the gear system. This determines the amount of power that the gear system consumes (mostly in the form of heat). and hence the horsepower requirements of the motor that drives the planetary.

Most designers would find it difficult to calculate the overall efficiency of a complex planetary. And when they derive the efficiency formula they cannot be sure that the formula is correct. In fact, the types with multiple carriers and split power, such as Ex 8, p 76, seldom have been analyzed in the literature.

This article, then, not only provides new easy-to-use "cook-book" formulas for overall efficiency and speed ratio, but also presents torque and angular speed values for the member-gears of the planetaries which are needed for the design of the gears themselves.

Planetary assembly...

and its schematic, torque-flow, fixed carrier and power-flow diagrams. The 20 systems that follow use the same notation system, and include speed-ratio formulas, R, and overall-efficiency formulas, E.



Such torque and speed values are also needed for the design of clutches or fastenings to lock the reaction gears from turning (the gears that do not rotate in the assembly).

How to use the boxed examples

The basis for deriving speed-ratio, efficiency, and torque values is given on p1-28. It will help you develop formulas for your own planetary arrangements should you need them. But chances are good that the planetary system you want is in the group of planetary systems that begin with Ex 1 below (there are 20 systems to select from). **Thus you can go directly to the boxed systems to obtain:**

Schematic diagrams displaying the skeletal structure of the various gears and planet carrier hook ups (see the illustrations below, left, for an explanation of the diagrams). The schematics also show the torque flow and power flow through all the members of the gear assembly.

Formulas for the overall speed ratio, R. This is a function of the number of teeth of the gears (N_1, N_2, etc). Typical values for the number of teeth have been selected in the examples to illustrate how torque and power splits or combines while flowing through the gears.

Formulas for overall efficiency, E. These formulas require knowing only the number of teeth and the efficiency between meshing teeth. For example, $\epsilon_{1,4}$ is the mesh efficiency between gears 1 and 4. You can assume a mesh efficiency of $\epsilon = 0.98$ (98%) for each external mesh and $\epsilon = 0.99$ for each internal mesh, in keeping with common practice. Because power losses are always *positive* in sign, the terms in the efficiency equations which constitute losses are frequently enclosed in bars to indicate that an absolute (positive) value is always to be used.

Table of torque and speed values for the various members of the system. Each member is listed in the first column. The letter *C* stands for carrier (see list of symbols on p1-28); the numbers represent gears.

The second column in the table shows the role that each member plays in the gear set. It may be the input or output member; it may be a reaction member, as is the case with the ring gear (3) in Example 1; or it may be an idler gear with no influence on the gear ratio.

The third column gives the torque acting on each member in relative terms. The torque on the input is set at 1, idlers have zero torque, and the torque on the other members is usually in terms of the speed ratio, *R*. Hence in Example 1, the torque on the output member is equal to $-R$, or for the number of teeth shown, $R = 1/5$, and the output torque will be 1/5 the input torque. The reaction member takes up 4/5 of the input torque. Input speed is assigned 1.

The fourth column gives the absolute angular speed, ω, of the member around its own axis (which includes the speed imparted to the member by a moving carrier.

The fifth column gives the angular speed of tooth engagement, ω_ϵ, which is its absolute speed minus the speed of the carrier.

By multiplying the *T* and ω values you obtain the power transmitted through a member. Also, by multiplying the *T* and ω_ϵ values you obtain the power circulating in its tooth engagement (if it is a gear).

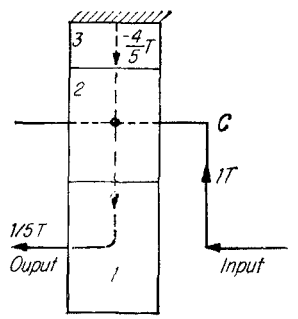
The boxed systems also display an "equivalent fixed carrier" power-flow diagram. By assuming that the carrier is fixed you quickly see a nondimensional, relative-value flow of torque, speed, and power, starting with assigned values of unity for input torque and input angular speed.

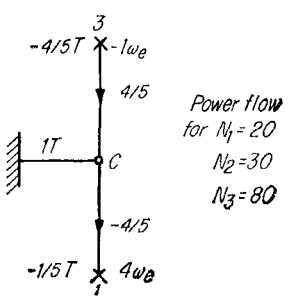
How to derive new formulas

The method which produced all the formulas and relationships in the boxed examples is based on the use of ratio factors, called *R*-factors,

Boxed formulas continue on next page
Derivation continues on page 1-28

1. Simple planetary system -- input to carrier





Speed-ratio formula

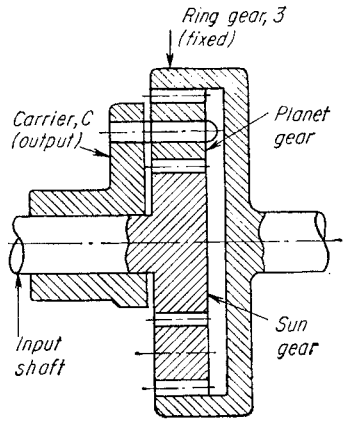
$$R = \frac{1}{1+\frac{N_3}{N_1}}$$

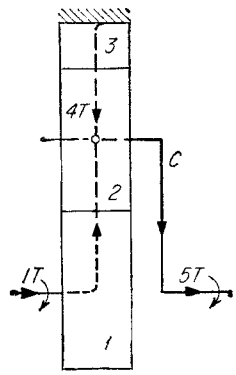
Efficiency formula

$$E = \frac{1}{1+\left[\frac{N_3}{N_1+N_3}\left(\frac{1}{\epsilon_{32}\,\epsilon_{21}}-1\right)\right]}$$

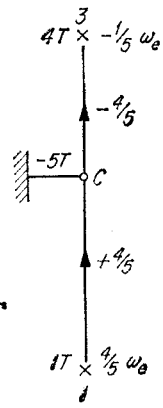
Member	Role	T	ω	ω_e
C	Input	1	1	–
1	Output	–R	1/5	$\omega-\omega_c$
		–1/5	5	4
3	Reaction	R–1	0	$\omega-\omega_c$
		–4/5		–1
2	Idler	0	–	$m_{21}\omega_{e1}$
				–8/3

2. Simple planetary system -- input to sun





Power & torque flow for $N_1 = 20$, $N_2 = 30$, $N_3 = 80$



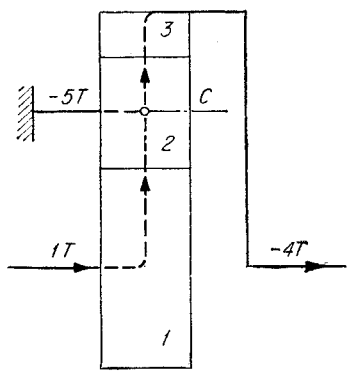
$$R = 1 + \frac{N_3}{N_1}$$

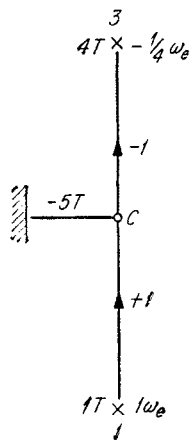
$$E = 1 - \left| \frac{N_3}{N_1 + N_3} (1 - \epsilon_{12}\,\epsilon_{23}) \right|$$

Member	Role	T	ω	ω_e
1	Input	1	1	$\omega - \omega_c$
C	Output	-R	1/R	—
3	Reaction	R-1	0	$\omega - \omega_c$
2	Idler	0	—	$m_{21} - \omega_{el}$

3. Simple planetary system -- with carrier fixed

Torque flow





Diagrams for $N_1 = 20$, $N_2 = 30$, $N_3 = 80$

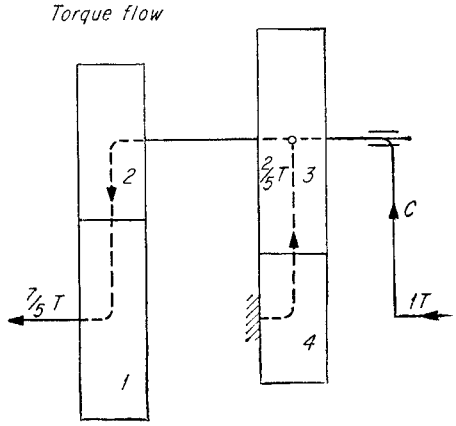
$$R = -\frac{N_3}{N_1}$$

$$E = \epsilon_{12}\,\epsilon_{23}$$

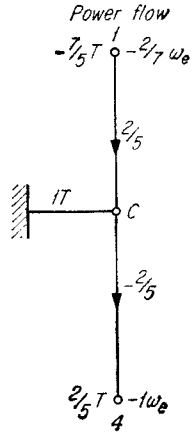
Member	Role	T	ω	ω_e
1	Input	1	1	$\omega - \omega_c$
3	Output	-R	1/R	$\omega - \omega_c$
C	Reaction	R-1	0	—
2	Idler	0	—	$m_{21} - \omega_{el}$

4. Compound planetary -- input to carrier

Torque flow



Power flow



Diagrams for $N_1 = 30$, $N_2 = 20$, $N_3 = 35$, $N_4 = 15$

$$R = \frac{1}{1 - \frac{N_2 N_4}{N_1 N_3}}$$

when $N_1 N_3 < N_2 N_4$

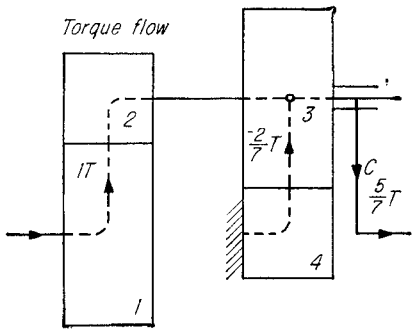
$$E = \frac{1}{1 + \left| \frac{-N_2 N_4}{N_1 N_3 - N_3 N_4} \right| \left(\frac{1}{\epsilon_{43}\,\epsilon_{21}} - 1 \right)}$$

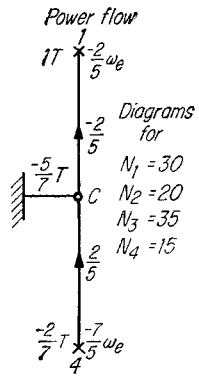
when $N_1 N_3 > N_2 N_4$

$$E = \frac{1}{1 + \left| \frac{-N_2 N_4}{N_1 N_3 + N_3 N_4} \right| (1 - \epsilon_{12}\,\epsilon_{34})}$$

Member	Role	T	ω	ω_e
C	Input	1	1	—
1	Output	-R	1/R	$\omega - \omega_c$
4	Reaction	R-1	0	$\omega - \omega_c$
2-3	Compound Idler	—	—	$m_{21} - \omega_{el}$

5. Compound planetary -- input to sun





$$R = 1 - \frac{N_2 N_4}{N_1 N_3}$$

When $N_1 N_3 < N_2 N_4$

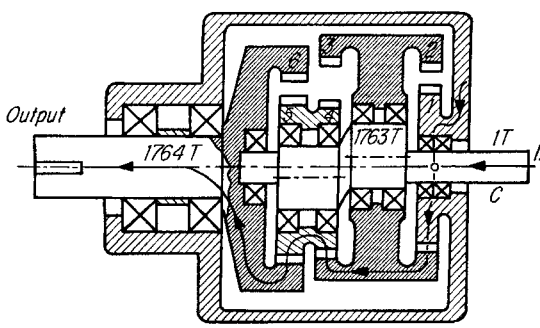
$$E = 1 - \left/ \frac{-N_2 N_4}{N_1 N_3 - N_2 N_4} \right/ \left(1 - \epsilon_{12}\,\epsilon_{34}\right)$$

When $N_1 N_3 > N_2 N_4$

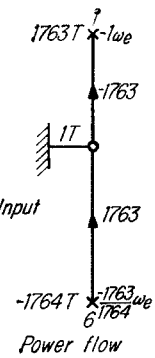
$$E = 1 - \left/ \frac{-N_2 N_4}{N_1 N_3 - N_2 N_4} \right/ \left(\frac{1}{\epsilon_{43}\,\epsilon_{21}} - 1\right)$$

Member	Role	T	ω	ω_e
1	Input	1	1	$\omega - \omega_c$
C	Output	-R	1/R	–
4	Reaction	R-1	0	$\omega - \omega_c$
2-3	Compound Idler	0	–	$m_{21}\omega_{e1}$

6. Double-eccentric planetary



For $N_1 = 82$ $N_2 = 84$ $N_3 = 86$
$N_4 = 82$ $N_5 = 82$ $N_6 = 84$
$R = 1764$

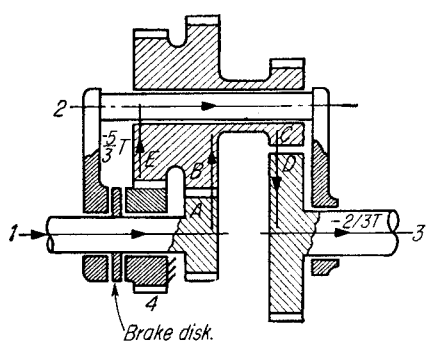


$$R = \frac{1}{1 - \frac{N_5 N_3 N_1}{N_6 N_4 N_2}}$$

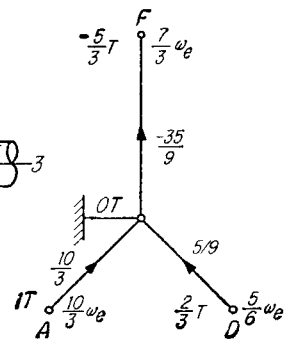
$$E = \frac{1}{1 + \left/ \frac{-N_5 N_3 N_1}{N_6 N_4 N_2 - N_5 N_3 N_1} \right/ \left(\frac{1}{\epsilon_{12}\,\epsilon_{34}\,\epsilon_{56}} - 1\right) \text{ if } N_6 N_4 N_2 < N_5 N_3 N_1 \left(1 - \epsilon_{65}\,\epsilon_{43}\,\epsilon_{21}\right) \text{ if } N_6 N_4 N_2 > N_5 N_3 N_1}$$

Member	Role	T	ω	ω_e
C	Input	1	1	–
6	Output	-R	1/R	$\omega - \omega_c$
1	Reaction	R-1	0	$\omega - \omega_c$
2,3	Compound Idler	0	–	$m_{21}\omega_{e1}$
4,5	Compound Idler	0	–	$m_{56}\omega_{e6}$

7. Triple-planet planetary



$$R = \frac{1 - (N_B N_F / N_A N_E)}{1 - (N_C N_F / N_D N_E)}$$



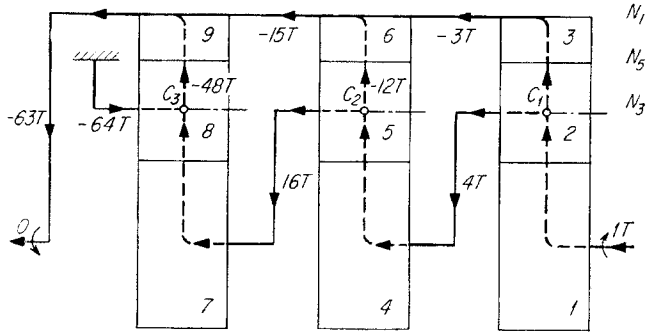
Member	Role	T	ω	ω_e
A	Input	1	1	$\omega - \omega_c$
D	Output	-R	1/R	$\omega - \omega_c$
F	Reaction	R-1	0	$\omega - \omega_c$
2 (Carrier)	Neutral	0	$\frac{N_A N_E}{N_A N_E - N_B N_F}$	–
E-B-C	Compound Idler	–		$m_{EF}\omega_{ef}$

$N_A = 20 = N_C$ $N_E = 35$
$N_B = 40 = N_D$ $N_F = 25$

$$E = \frac{1 - \left/ \frac{-N_B N_F}{N_A N_E - N_B N_F} \right/ \left(1 - \epsilon_{AB}\,\epsilon_{EF}\right) \text{ if } N_A N_E < N_B N_F \left(\frac{1}{\epsilon_{FE}\,\epsilon_{BA}} - 1\right) \text{ if } N_A N_E > N_B N_F}{1 + \left/ \frac{-N_C N_F}{N_D N_E - N_C N_F} \right/ \left(\frac{1}{\epsilon_{FE}\,\epsilon_{CD}} - 1\right) \text{ if } N_D N_E < N_C N_F \left(1 - \epsilon_{DC}\,\epsilon_{EF}\right) \text{ if } N_D N_E > N_C N_F}$$

8. Triple ring-gear drive

$$R=(1+\frac{N_3}{N_1})\left[(1+\frac{N_6}{N_4})(-\frac{N_9}{N_7})-\frac{N_6}{N_4}\right]-\frac{N_3}{N_1}$$



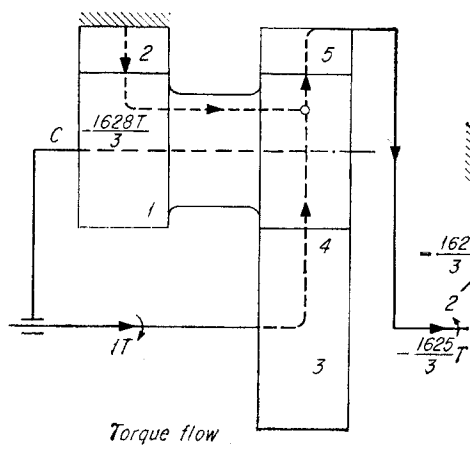
Torque flow numbers for
$N_1 = N_2 = N_4 = 20$
$N_5 = N_7 = N_8 = 20$
$N_3 = N_6 = N_9 = 60$

Member	Role	T	ω
1	Input	1	1
10	Output	-R	1/R
C_3	Reaction	R-1	0
3	Split output from 1	$-m_{13}$	ω_0
C_1		$-(1-m_{13})$	ω_4
4	Input to 2	$-T_{c1}$	$\omega_e+\omega_{e2}$

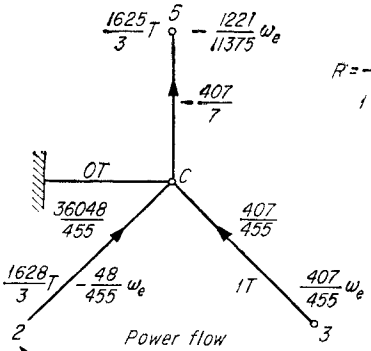
Member	Role	T	ω
6	Split output from 2	$-(1-m_{13})m_{46}$	ω_0
C_2		$T_{c1}(1-m_{46})$	ω_7
7	Input to (3)	$-T_{c2}$	ω_e
9	Output from (3)	$-T_7\, m_{73}$	ω_0
2	Idler	0	$m_{21}\omega_{e1}$
5	Idler	0	$m_{54}\omega_{e4}$
8	Idler	0	$m_{87}\omega_{e7}$

$$E=1-\frac{16}{21}(1-\epsilon_{12}\,\epsilon_{23})-\frac{16}{21}(1-\epsilon_{45}\,\epsilon_{56})-\frac{16}{21}(1-\epsilon_{78}\,\epsilon_{89})$$

9. Minuteman-cover drive



Torque flow



Power flow

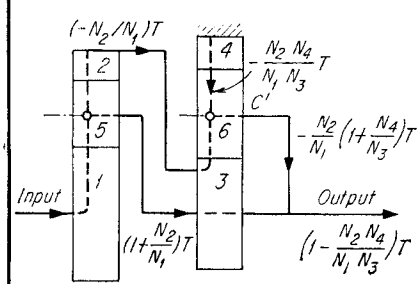
$$R=\frac{1+\frac{N_4 N_2}{N_3 N_1}}{1-\frac{N_4 N_2}{N_5 N_1}}=-541\tfrac{2}{3} \text{ for } N_1=32,\ N_2=74,\ N_3=9,\ N_4=33,\ N_5=75$$

Member	Role	T	ω	ω_e
3	Input	1	1	$\omega-\omega_c$
5	Output	-R	1/R	$\omega-\omega_c$
2	Reaction	R-1	0	$\omega-\omega_c$
C	Neutral	0	$-\omega_c=\frac{N_3 N_1}{N_4 N_2+N_3 N_1}$	
1 – 4	Idler	0	—	$m_{43}\omega_{e3}$

$$E=\frac{1-\left|\left[N_4 N_2/(N_3 N_1+N_4 N_2)\right](1-\epsilon_{34})\right|}{1+\left|\left[\frac{-N_4 N_2}{N_5 N_1-N_4 N_2}-\frac{N_4 N_2}{N_3 N_1+N_4 N_2}\right]\left(\frac{1}{\epsilon_{21}\epsilon_{45}}-1\right)\right|+\left|\frac{N_4 N_2}{N_3 N_1+N_4 N_2}\left(\frac{1}{\epsilon_{45}}-1\right)\right|}$$

Coupled planetary drives

10.

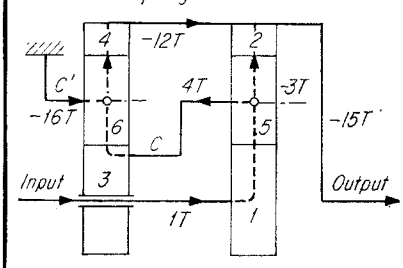


$$R=1-\frac{N_2 N_4}{N_1 N_3}$$

$$E=1-\left|\frac{-N_2 N_4}{N_1 N_3-N_2 N_4}(1-\epsilon_{15}\epsilon_{52}\epsilon_{36}\epsilon_{64})\right|$$

11.

$N_2 = N_4 = 60$
$N_1 = N_3 = 20$

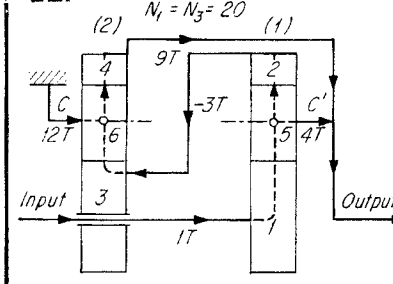


$$R=(1+\frac{N_2}{N_1})(-\frac{N_4}{N_3})-\frac{N_2}{N_1}$$

$$E=1-\left|\frac{4}{5}(1-\epsilon_{15}\epsilon_{52})\right|-\left|\frac{4}{5}(1-\epsilon_{36}\epsilon_{64})\right|$$

12.

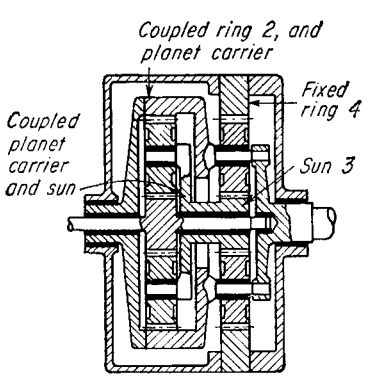
$N_2 = N_4 = 60$
$N_1 = N_3 = 20$

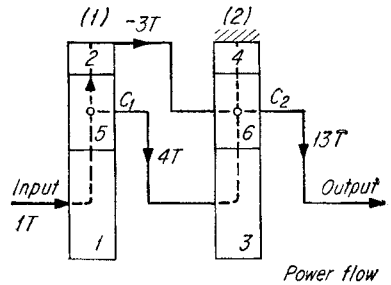


$$R=1+\frac{N_2}{N_1}(1+\frac{N_4}{N_3})$$

$$E=1-\left|\frac{12}{13}(1-\epsilon_{15}\,\epsilon_{52})\right|-\left|\frac{9}{13}(1-\epsilon_{36}\,\epsilon_{64})\right|$$

13. Coupled planetary drives (continued)



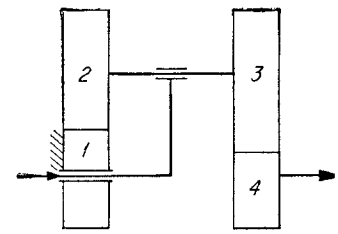


$$R = 1 + \frac{N_4}{N_3}\left(1 + \frac{N_2}{N_1}\right)$$

Power flow for $N_2 = N_4 = 60$, $N_1 = N_3 = 20$

$$E = 1 - \left|\tfrac{3}{4}(1 - \epsilon_{15}\,\epsilon_{52})\right| - \left|\tfrac{12}{13}(1 - \epsilon_{36}\,\epsilon_{64})\right|$$

14. Differential drive

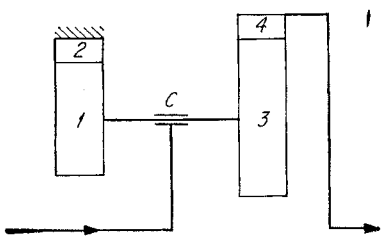


$$R = \frac{1}{1 - \frac{N_3 N_1}{N_4 N_2}}$$

$$E = 1 \Big/ \left[1 + \left|\frac{-N_3 N_1}{N_4 N_2 - N_3 N_1}(1 - \epsilon_{43}\,\epsilon_{21})\right|\right]$$

Fixed differential drives

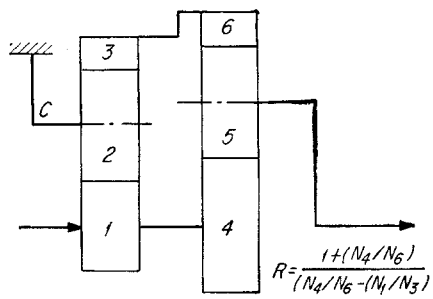
15.



$$R = \frac{1}{1 - \frac{N_3 N_2}{N_4 N_1}}$$

$$E = \frac{1}{1 + \left|\frac{-N_3 N_1}{N_4 N_2 - N_3 N_1}\right| \begin{matrix} \left|\left(\frac{1}{\epsilon_{21}\,\epsilon_{43}} - 1\right)\right| \text{ if } N_3 N_2 > N_4 N_1 \\ \left|(1 - \epsilon_{43}\,\epsilon_{21})\right| \text{ if } N_3 N_2 < N_4 N_1 \end{matrix}}$$

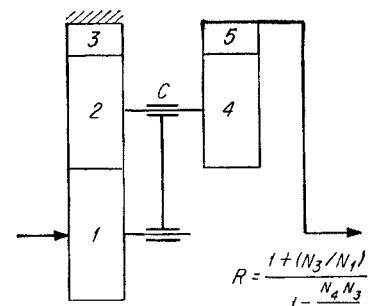
16.



$$R = \frac{1 + (N_4/N_6)}{(N_4/N_6) - (N_1/N_3)}$$

$$E = \frac{1 - \left|\frac{N_6}{N_4 + N_6}(1 - \epsilon_{45}\,\epsilon_{56})\right|}{1 + \left|\frac{-N_6 N_1}{N_4 N_3 - N_6 N_1}\right| \begin{matrix} \left|\left(\frac{1}{\epsilon_{12}\,\epsilon_{23}\,\epsilon_{15}\,\epsilon_{54}} - 1\right)\right| \text{ if } N_6 N_4 > N_4 N_3 \\ \left|(1 - \epsilon_{45}\,\epsilon_{56}\,\epsilon_{32}\,\epsilon_{21})\right| \text{ if } N_6 N_4 < N_4 N_3 \end{matrix}}$$

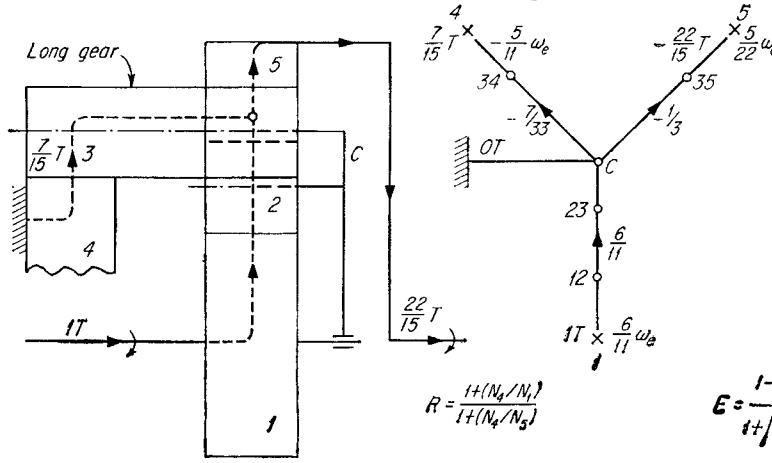
17.



$$R = \frac{1 + (N_3/N_1)}{1 - \frac{N_4 N_3}{N_5 N_2}}$$

$$E = \frac{1 - \left|\frac{N_3}{N_1 - N_3}\left(\frac{1}{\epsilon_{32}\,\epsilon_{21}} - 1\right)\right|}{1 + \left|\frac{-N_4 N_3}{N_5 N_2 - N_4 N_3}\right| \begin{matrix} \left|\left(\frac{1}{\epsilon_{32}\,\epsilon_{45}} - 1\right)\right| \text{ if } N_4 N_3 > N_5 N_2 \\ \left|(1 - \epsilon_{54}\,\epsilon_{23})\right| \text{ if } N_4 N_3 < N_5 N_2 \end{matrix}}$$

18. Fordomatic transmission planetary



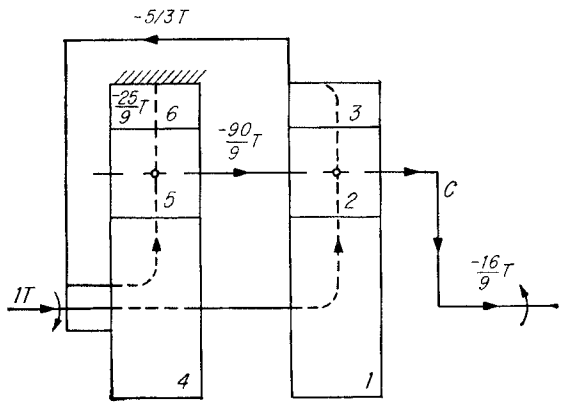
$$R = \frac{1 + (N_4/N_1)}{1 + (N_4/N_5)}$$

Power flow for:
$N_1 = 30$, $N_2 = N_3 = 18$, $N_4 = 36$, $N_5 = 72$

Member	Role	T	ω	ω_e
1	Input	1	1	$\omega - e$
5	Output	-R	$1/R$	$\omega - c$
4	Reaction	R-1	0	$\omega - \omega_c$
C	Neutral	0	$\frac{1}{1 + (N_4/N_1)}$	—
2	Idler	0		$m_{21}\omega_{e1}$
3	Idler	0		$m_{35}\omega_{e5}$

$$E = \frac{1 - \left|\left(\frac{N_4}{N_1 + N_4} - \frac{N_4}{N_5 + N_4}\right)(1 - \epsilon_{12}\,\epsilon_{23}\,\epsilon_{34})\right| - \left|\frac{N_4}{N_5 + N_4}(1 - \epsilon_{12}\,\epsilon_{23})\right|}{1 + \left|\frac{N_4}{N_5 + N_4}\left(\frac{1}{\epsilon_{35}} - 1\right)\right|}$$

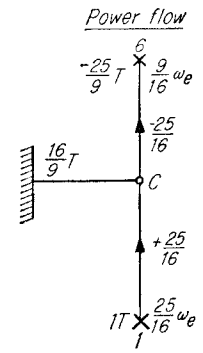
19. GM Hydramatic planetary



$N_1 = 45 = N_4$

$N_2 = 15 = N_5$

$N_3 = 75 = N_6$



$$R = -\frac{N_3}{N_1}\left(1+\frac{N_6}{N_4}\right)+\left(1+\frac{N_3}{N_1}\right)$$

$$E = 1-\left|\frac{-N_3 N_6}{N_1 N_4 - N_3 N_6}(1-\epsilon_{12}\,\epsilon_{23}\,\epsilon_{45}\,\epsilon_{56})\right|$$

SYMBOLS

C = carrier (also called "spider")—a non-gear member of a gear train whose rotation affects gear ratio
m = fixed gear ratio (for a train with no rotating carriers)
N = number of teeth
R = overall speed reduction ratio = ω_i/ω_o
f = follower (in a gear train)
d = driver (in a gear train)
t = gear train
ω = angular speed
T = torque
ϵ = efficiency between meshing teeth
E = overall planetary gear mesh efficiency

Subscripts
o = output
i = input
r = reaction
c = carrier
e = equivalent
1, 2, 3, etc.— gears in a train

Use of double subscripts
m_{13} = gear ratio for the train starting at gear *1* as input and proceeding to gear *3* as output
ϵ_{13} = mesh efficiency between gears *1* and *3*

which give the ratio between individual members of the assembly in terms of m, the gear ratio for the assembly if the carriers are assumed fixed.

When carrier is input

$$R_i = \frac{1}{1 - m_{or}}$$

When carrier is output

$$R_o = 1 - m_{ir}$$

When carrier is a reaction member (locked to frame)

$$R_r = m_{io}$$

Subscript *or* refers to the ratio between output (o) and reaction (r) members, etc. See list of symbols for definition of other symbols and subscripts.

Speed ratio formulas

The R-factors lead directly to the speed ratio formulas. For example, for a complicated planetary with five gears, the Minuteman cover drive, Example 9, in which one ring gear is fixed and the second is the output, the overall ratio is

$$R = R_o R_i = \frac{1 - m_{ir}}{1 - m_{or}}$$

1) Write out the m-train sequence, as shown after Step 6 below.

2) Place a plus sign over the number of the first driver.

3) Mentally turn the first driver and note the direction of rotation of the next gear in mesh with it.

4) Place the appropriate sign over the number of this next gear.

5) Continue in this manner to the end of the train.

6) Signs of m are the same as that of the last follower.

$$t_{ir} = \begin{matrix}(+) & (-) & (-) & (-)\\ 3 & - & 4 & - & 1 & - & 2\\ (d) & & (f) & & (d) & & (f)\end{matrix}$$

$$t_{or} = \begin{matrix}(+) & (+) & (+) & (+)\\ 5 & - & 4 & - & 1 & - & 2\\ (d) & & (f) & & (d) & & (f)\end{matrix}$$

7) Magnitude of m = product of N_f's/product of N_d's.

$$m_{ir} = m_{32} = N_4 N_2 / N_3 N_1$$
$$m_{or} = m_{52} = N_4 N_2 / N_5 N_1$$

$$R = \frac{1-\left(-\dfrac{N_4 N_2}{N_3 N_1}\right)}{1-\dfrac{N_4 N_2}{N_5 N_1}} = \frac{1+\dfrac{N_4 N_2}{N_3 N_1}}{1-\dfrac{N_4 N_2}{N_5 N_1}}$$

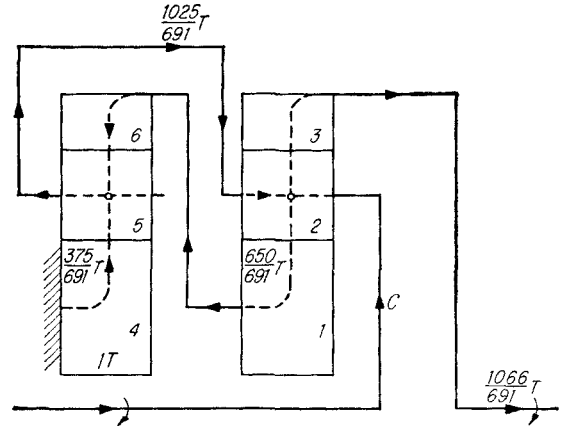
Thus for the number of teeth selected (see Example 9) you obtain a 541⅔ speed reduction. The final formulas derived by this method are given in the boxed examples.

Efficiency formulas

The overall gear-mesh efficiency formula for planetary gear systems is

$$E = \frac{1 - \text{Total } R_o \text{ and } R_r \text{ losses}}{1 + \text{Total } R_i \text{ losses}}$$

20. Tractor transmission planetary



$N_1 = 50$

$N_2 = 17 = N_5$

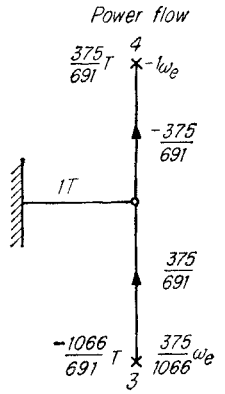
$N_3 = 82$

$N_4 = 45$

$N_6 = 78$

$$R = \frac{1}{1 - \frac{N_1 N_4}{N_3 N_6}}$$

$$E = \frac{1}{1 + \left| \frac{-N_6 N_1}{N_3 N_4 - N_6 N_1} \left(1 - \epsilon_{32}\epsilon_{21}\epsilon_{65}\epsilon_{54}\right) \right|}$$



with their mates. These losses can be determined by first computing "ρ factors" from the following equations:

$$\rho_i = \frac{m_{or} - m_{or}}{1 - m_{or}}$$

$$\rho_o = \frac{-m_{ir}}{1 - m_{ir}}$$

Then, in computing R_i loss, when ρ_i comes out positive, use the equation

$$R_i \text{ loss} = \rho_i \left[\frac{1}{\epsilon_{ro}} - 1 \right]$$

When ρ_i is a negative value, use

$$R_i \text{ loss} = |\rho_i| (1 - \epsilon_{or})$$

Note that you employ the absolute value of ρ_i in the second equation (you assume that it is a positive value).

In the same manner, when ρ_o is positive

$$R_o \text{ loss} = \rho_o (1 - \epsilon_{ir})$$

When ρ_o is negative

$$R_o \text{ loss} = |\rho_o| \left[\frac{1}{\epsilon_{ri}} \right] - 1$$

Also, in all cases

$$R_r \text{ loss} = 1 - \epsilon_{io}$$

The ρ-factor is a "signed" quantity. Its sign indicates the direction of power flow in the equivalent fixed carrier (EFC) train (power flow diagram), and its size indicates the magnitude of the equivalent power. In the power flow diagram the law of direct-current flow for electrical circuits applies. The flow to a junction point in the power flow diagram must be equal to the flow from the junction point.

As can be noted in Fig 1, input is to gear 1 and output is from carrier C with gear 3 fixed (reaction member). Equivalent power, P_e, flows from 1 to 2 through gear meshes 12 and 23, and input power carries a plus sign and output power a minus sign. The power losses sustained in driving these gears through mesh with their mates in the EFC train is the same as (equivalent to) the power losses circulating in the same tooth engagements in the train.

The external torque, T, acting on each member remains the same in the EFC train, but the speed, ω, of each member is reduced algebraically by the speed of the carrier, ω_c (since the carrier is artificially stopped). Therefore $\omega - \omega_c = \omega_e$ or *equivalent speed.*

Turn now to Example 1, where a specific planetary gear set with numbers of teeth given is shown. Here gear 3 is fixed, the EFC train schematic is omitted, and the EFC power flow is shown, this time with actual torque and equivalent speed values shown on the EFC power flow diagram corresponding to T and ω_e values in the table.

The ρ_i factor for Example 1 is

$$\rho_i = \frac{-m_{13}}{1 - m_{13}} = \frac{N_3}{N_1 + N_3}$$

If $N_1 = 20$, $N_2 = 30$ and $N_3 = 80$, then $\rho_i = +4/5$. The plus sign means that the equivalent power flow is "like" the flow in the planetary system, and 4/5 means that the equivalent power is 4/5 of the planetary transmitted power. The term "like" is carefully chosen, because the flow is not identical. In the planetary train the transmitted power flows from i to o or C to 1. In the EFC train equivalent power flows from r to o or 3 to 1. The flow is "like" because in both cases the flow is *to o.* (Had the sign of ρ_i been minus then the flow would have been *from o* to r and therefore "unlike".)

Returning to the equation for efficiency, it can be rewritten as

$$E = \frac{1}{1 + |\rho_i| \left(\frac{1}{\epsilon_{ro}} - 1 \right)}$$

This equation is then put into the form you see in Ex 1, with the number of teeth replacing ρ. This is done for every example for ease of use.

To better understand the flow of power in a planetary, it is helpful to know that the equivalent speed of a member is equal to its speed of tooth engagement. Equivalent power and power circulating in tooth engagement likewise are identical, since power is proportional to the product of torque and speed. This means that frictional power losses due to gear tooth action in planetary trains and power losses in EFC trains are the same and are equal to the difference between equivalent input and output power.

Checklist for Planetary-Gear Sets

These five tests quickly tell whether the gears will mesh, and whether there is room for them to fit together.

Hugh P. Hubbard

SYMBOLS

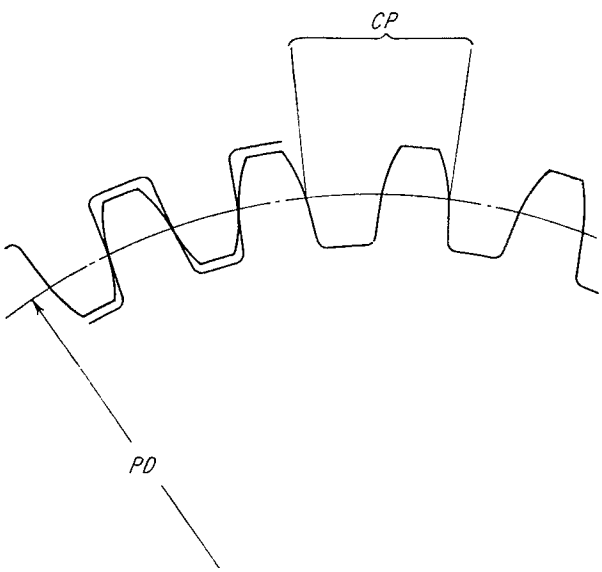
CP	= Circular pitch, in.
L	= Distance from center of sun gear to center of planet gear, in.
M	= Major or outside diameter of gear, in.
DP	= Diametral pitch, teeth/in.
m	= Minor, or working depth diameter of gear, in.
N	= Number of teeth per gear
PD	= Pitch diameter, in.
x	= The whole number in dividend when N_s is divided by number of planets
y	= The whole number in dividend when N_r is divided by number of planets
z	= Increment for locating planet gear
α	= Angular location of planet gear

You have decided to design a planetary-gear system with a certain gear ratio, and have chosen the number of teeth for each gear to get that ratio. Will it work? Will the gears fit together to make a workable system?

If they can pass the following five tests, they will.

1—Do all gears have the same circular pitch?

If they do not, the gears will not mesh.



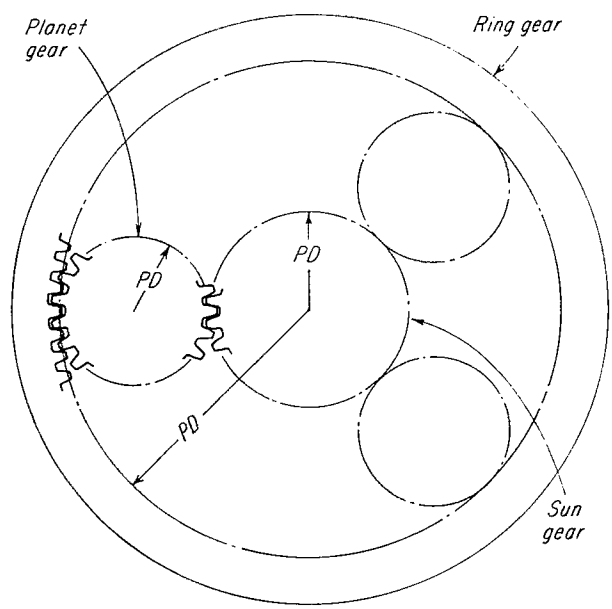
Circular pitch $CP = \pi/DP = PD/N$

Circular pitch and number of teeth determine pitch diameter, which leads to the next test:

2—Will the gears mate at the pitch diameters?

This equation shows whether the planet gear will fill the space between the sun gear and the ring gear:

$$N_p = \frac{N_r - N_s}{2}$$



3—Will the teeth mesh?

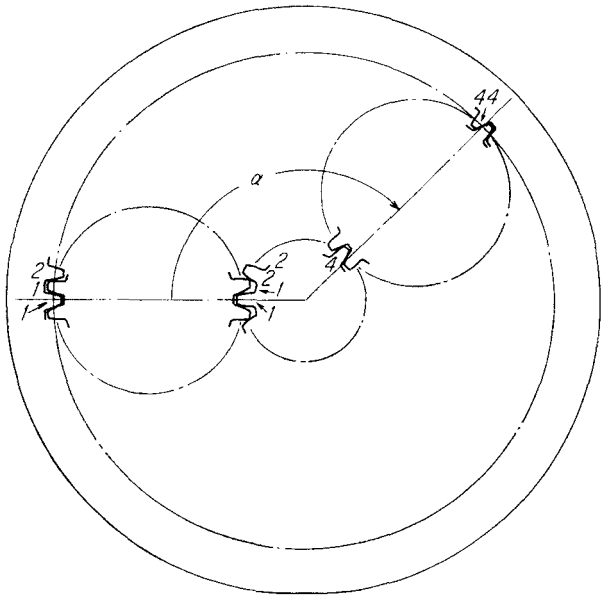
Gears that pass the first two tests will not necessarily pass this one. If the gears have the wrong number of

teeth, the planet gear will not mesh with the sun gear and the ring gear at the same time. Gears with numbers of teeth divisible by three will mesh. There are two other possible cases.

Case I—The number of teeth on the sun gear divides evenly into the number of teeth on the ring gear. This set will mesh, if allowance is made by spacing the planet gears unevenly around the sun gear.

EXAMPLE: In a set of planetary gears the ring gear has 70 teeth, the sun gear 14 teeth and each of the three planet gears 28 teeth. Even spacing would place the planet gears every 120°, but in this case they must be placed slightly to one side of the 120° point to mesh. Since N_s divides evenly into N_r there is a tooth on the ring gear opposite every tooth on the sun gear. Therefore, it is possible to fit a planet gear opposite any tooth on the sun gear. Tooth 6, five circular pitches from tooth 1, is the choice because it is closest to being one-third of the way around. It is opposite tooth 26 on the ring gear, because $Nr/Ns = 70/14 = 25/5$.

Case II—The number of teeth on the sun gear does not divide evenly into the number of teeth on the ring gear. This set may or may not mesh; the following example shows how to tell.



EXAMPLE: In a set of planetary gears with three planets, the ring gear has 134 teeth, the sun gear 14 and the planet gears 60 each. $N_s/3 = 14/3 = 4.67$, so the whole number $x = 4$. $N_r/3 = 134/3 = 44.67$, so the whole number $y = 44$.

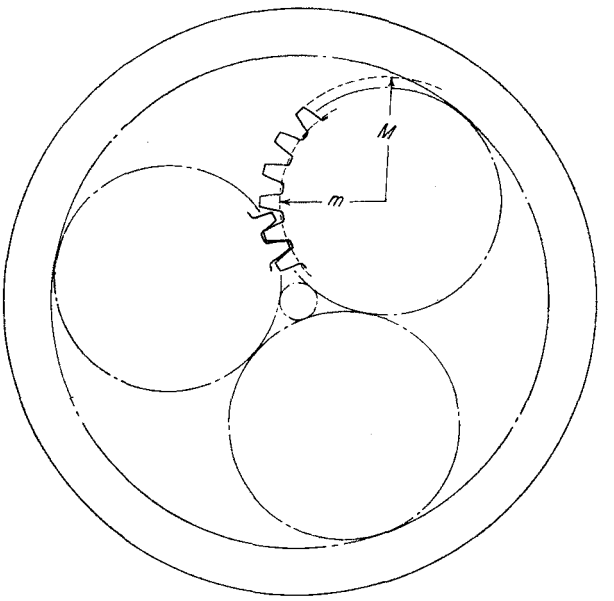
Plug these numbers into the locating equation

$$(x+z)\,N_r/N_s = y+(1-z) = (4+z)\,134/14 = 44+(1-z)$$
$$10.57\,z = 6.72$$
$$z = 0.636$$

Location of the planet gear as a fractional part of the circular distance around the set is $(x + z)/N_s = 4.636/14 = 0.3311$, and $y + (1 - z)/N_r = 44.364/134 = 0.3311$. The answers agree to four places, so the gears will mesh. If the answers don't agree to four places, there will be interference.

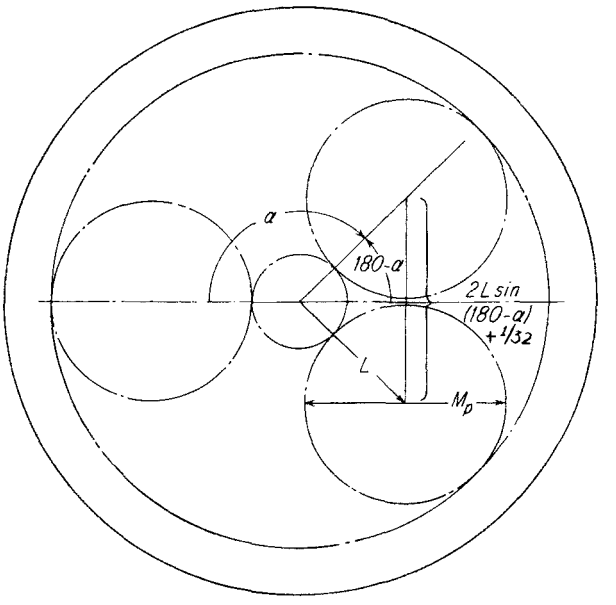
Angle $a = 0.3311 \times 360 = 119.2°$

4—Can three planets fit around the sun gear?



They will if the major diameters adhere to the limitation $M_p + m_s/2 < m_r$ by a safety clearance of 1/32 in. more than maximum tolerances.

5—Will irregularly spaced planets hit each other?

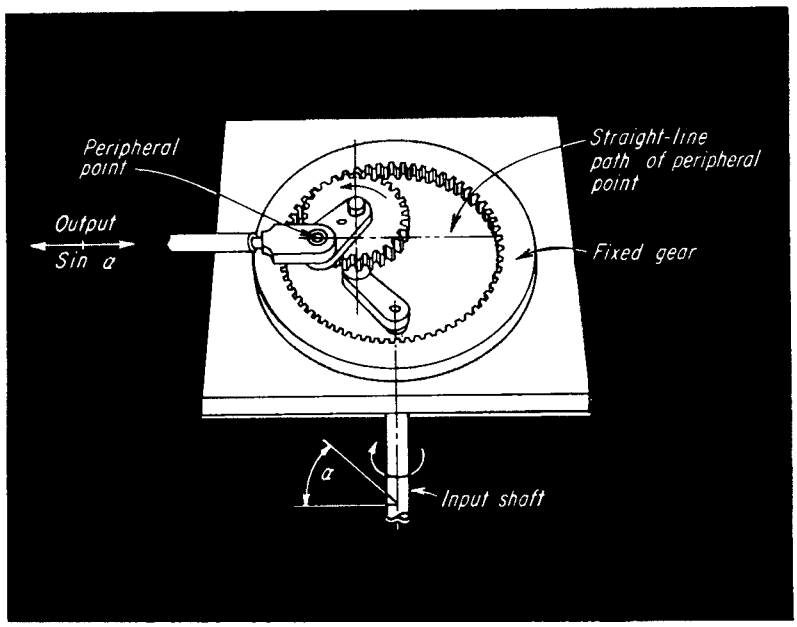


Two adjacent planets will not hit each other if $2L \sin (180 - a) > M_p + 1/32$ in. safety clearance. Sun-to-planet center-to-center distance $L = (PD_s + PD_p)/2$. ■

Cardan-Gear Mechanisms

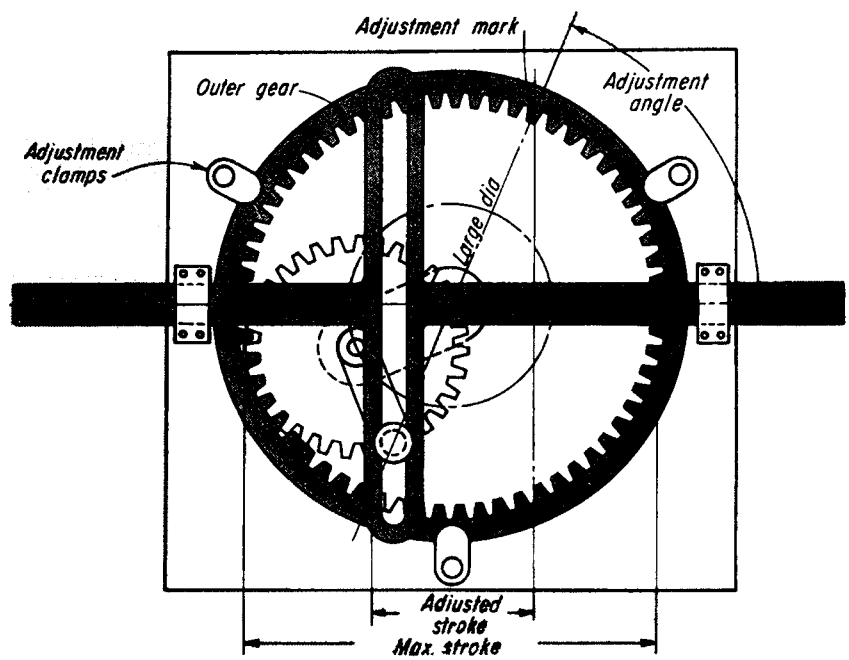
These gearing arrangements convert rotation into straight-line motion, without need for slideways.

Sigmund Rappaport



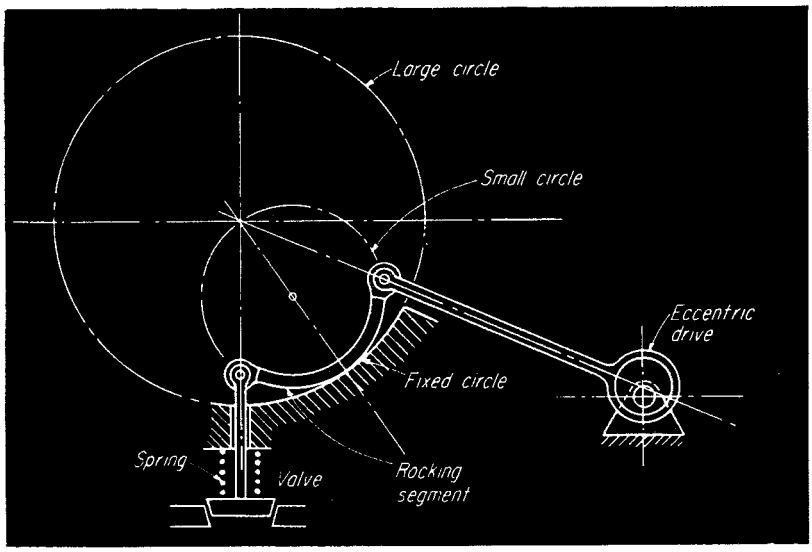
1

Cardan gearing . . . works on the principle that any point on the periphery of a circle rolling on the inside of another circle describes, in general, a hypocycloid. This curve degenerates into a true straight line (diameter of the larger circle) if diameters of both circles are in the ratio of 1:2. Rotation of input shaft causes small gear to roll around the inside of the fixed gear. A pin located on pitch circle of the small gear describes a straight line. Its linear displacement is proportional to the theoretically true sine or cosine of the angle through which the input shaft is rotated. Among other applications, Cardan gearing is used in computers, as a component solver (angle resolver).



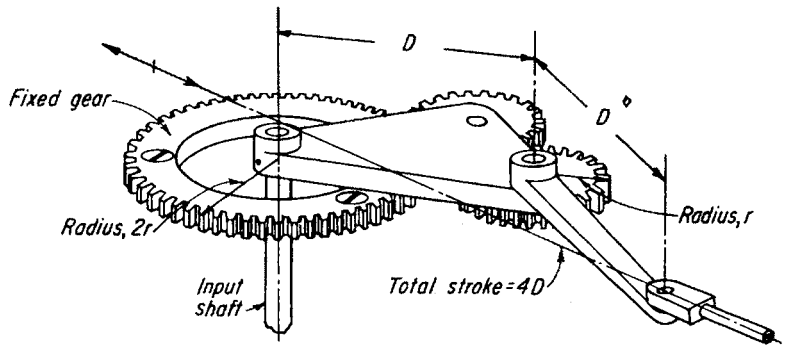
2

Cardan gearing and Scotch yoke . . . in combination provide an adjustable stroke. Angular position of outer gear is adjustable. Adjusted stroke equals the projection of the large dia, along which the drive pin travels, upon the Scotch-yoke centerline. Yoke motion is simple harmonic.



3

Valve drive . . . exemplifies how Cardan principle may be applied. A segment of the smaller circle rocks to and fro on a circular segment whose radius is twice as large. Input and output rods are each attached to points on the small circle. Both these points describe straight lines. Guide of the valve rod prevents the rocking member from slipping.

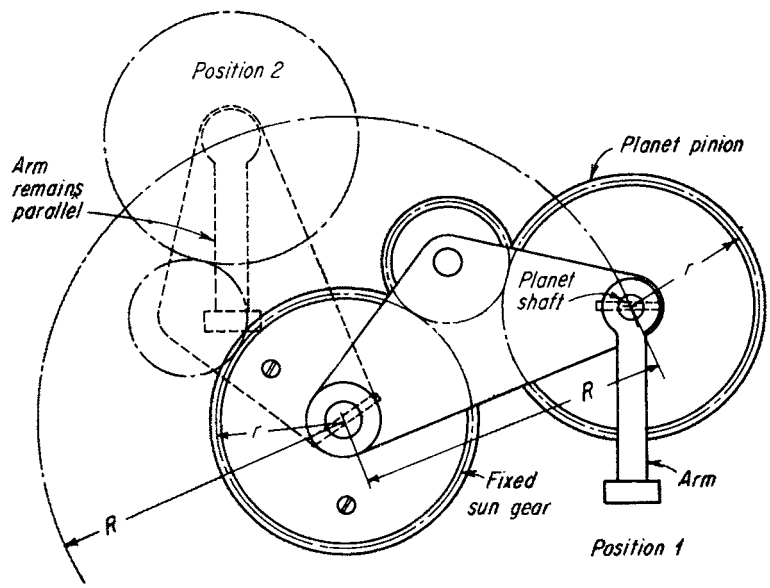


4

Simplified Cardan principle . . . does away with need for the relatively expensive internal gear. Here, only spur gears may be used and the basic requirements should be met, i.e. the 1:2 ratio and the proper direction of rotation. Latter requirement is easily achieved by introducing an idler gear, whose size is immaterial. In addition to cheapness, this drive delivers a far larger stroke for the comparative size of its gears.

5

Rearrangement of gearing . . . in (4) results in another useful motion. If the fixed sun-gear and planet pinion are in the ratio of 1:1, then an arm fixed to the planet shaft will stay parallel to itself during rotation, while any point on it describes a circle of radius *R*. An example of application: in conjugate pairs for punching holes on moving webs of paper.



Geneva Wheel Design

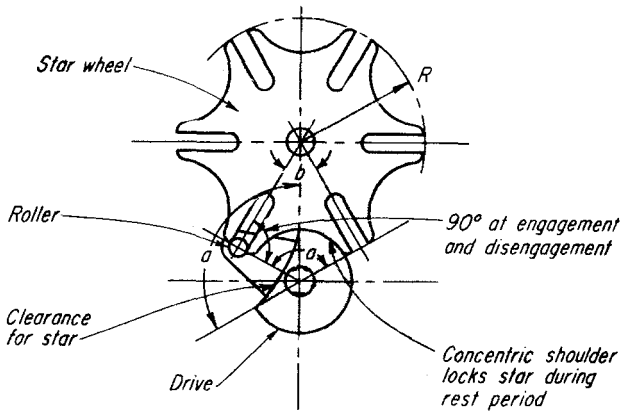
These diagrams and formulas will help.

Douglas C. Greenwood

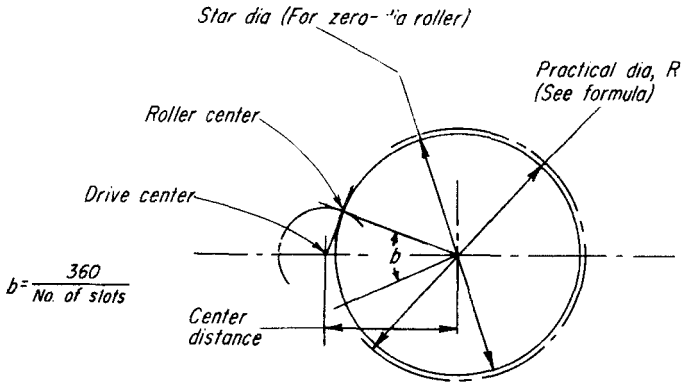
The Geneva wheel was originally used as a stop for preventing overwind in watch springs—leave one of the slot positions unslotted and the number of turns the drive can make is limited. Now, the Geneva wheel is one of the most useful mechanisms for providing high-speed, intermittent, rotary motion.

In the position shown in the first diagram the roller is on the point of entering the slot and about to begin driving the star wheel. Drive ceases when the roller has moved through angle a. The star is locked during its rest periods because the concentric shoulder of the driving member has engaged the corresponding edge of the star. To permit the drive wheel to continue turning it must have a recess for star clearance.

When a Geneva mechanism is used in a machine, the machine speed depends upon the speed at which the star can be driven. Although, on balance, no advantage is gained, some modifications have been investigated that reduce the maximum acceleration and the resulting force. If, for example, the slots are not exactly tangential to the crank circle at their entrance, acceleration forces are reduced. Drive speed can, therefore, be increased. Such design changes, while decreasing acceleration, increase deceleration. Also, hammering will occur at the concentric surface, with a resultant noisy operation.



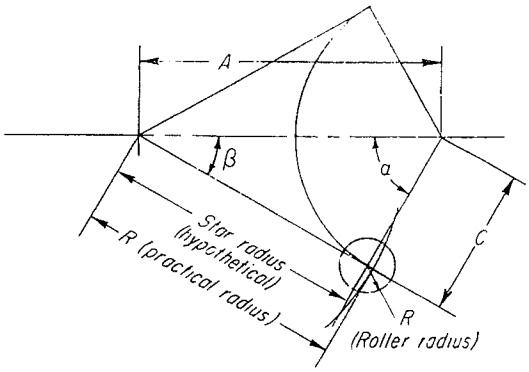
Complete Geneva Mechanism

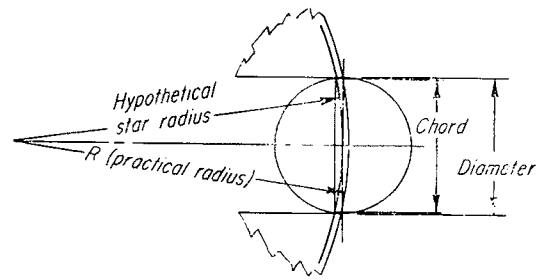


Basic step in layout

Geneva mechanism . . .

can theoretically have from 3 to any number of slots. In practice from 4 to 12 is enough to cover most requirements. Once the number of slots and crank dia are known, the layout can be started by constructing a right-angled triangle on the center distance. A tangent to the star dia where it is cut by a leg of the angle b is the crank radius. The clearance angle on the drive is 180 minus b. Layout is completed by drawing the rest of the mechanism outline. Roller and slot dimensions will depend on forces to which the mechanism will be subjected.





DESIGN FORMULAS

Values such as angular position at any instant, velocity, acceleration and practical star-wheel dia can be calculated from the formulas that follow. Illustrated in the sketch above are symbols that appear in the formulas.

Hypothetical star radius . . .

would be practical only for a roller of zero dia. Otherwise *R* will always be the greater radius. This ensures that correct roller and slot clearance is maintained at entrance and exit. Difference between chord and dia would be the excess clearance that would result (at time of entry and exit) from using incorrect radius.

Center distance A

$$= CM \text{ where } M = \frac{1}{\text{Sin } \frac{180}{\text{no. of slots}}}$$

Angular displacement:

$$\text{Tan } \beta = \frac{\text{Sin } \alpha}{M - \cos \alpha}$$

Angular velocity

$$= \frac{d\beta}{dt}$$

$$= \omega \left(\frac{M \cos \alpha - 1}{1 + M^2 - 2M \cos \alpha} \right)$$

where Ω = radians/sec

Angular acceleration, radians/sec²

$$= \frac{d^2\beta}{dt^2}$$

$$= \omega^2 \left(\frac{M \sin \alpha \, (1 - M^2)}{(1 + M^2 - 2M \cos\alpha)^2} \right)$$

Max acceleration occurs when

$$\cos\alpha = \pm \sqrt{\left[\left(\frac{1 + M^2}{4M} \right)^2 + 2 \right]} - \left(\frac{1 + M^2}{4M} \right)$$

Max acceleration and therefore max wear occurs at about ⅓ to ¼ distance down the slot length from the wheel edge.

Practical star-wheel dia

$$2R = \sqrt{A^2 - C^2 + \mu^2}$$

ANGULAR DISPLACEMENT OF DRIVE
during periods of star-wheel rest and motion

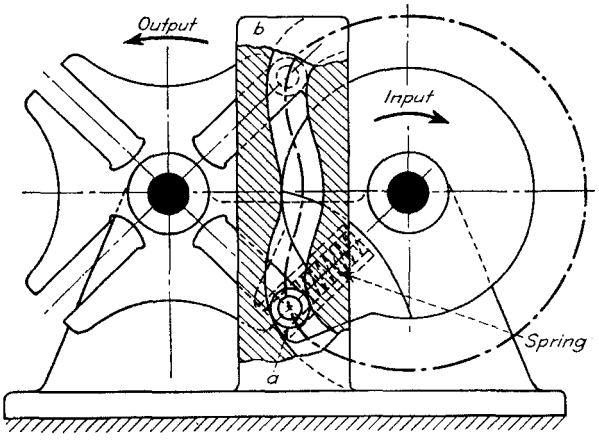
Slot angle	No. of drive rollers	Drive crank: Degrees of driving stroke (motion period)	Drive crank: Degrees of free turning (dwell period)
90° 4 slot	1	90	270
	2	180	180
	3	270	90
	4	360	0
72° 5 slot	1	108	252
	2	216	144
	3	324	36
60° 6 slot	1	120	240
	2	240	120
45° 8 slot	1	135	225
	2	270	90
36° 10 slot	1	144	216
	2	288	72
30° 12 slot	1	150	210
	2	300	60

Modified Geneva Drives and Special Mechanisms

These sketches were selected as practical examples of uncommon, but often useful mechanisms. Most of them serve to add a varying velocity component to the conventional Geneva motion.

Sigmund rappaport

Fig. 1—(Below) In the conventional external Geneva drive, a constant-velocity input produces an output consisting of a varying velocity period plus a dwell. In this modified Geneva, the motion period has a constant-velocity interval which can be varied within limits. When spring-loaded driving roller *a* enters the fixed cam *b*, the output-shaft velocity is zero. As the roller travels along the cam path, the output velocity rises to some constant value, which is less than the maximum output of an unmodified Geneva with the same number of slots; the duration of constant-velocity output is arbitrary within limits. When the roller leaves the cam, the output velocity is zero; then the output shaft dwells until the roller re-enters the cam. The spring produces a variable radial distance of the driving roller from the input shaft which accounts for the described motions. The locus of the roller's path during the constant-velocity output is based on the velocity-ratio desired.



Saxonian Carton Machine Co., Dresden, Germany

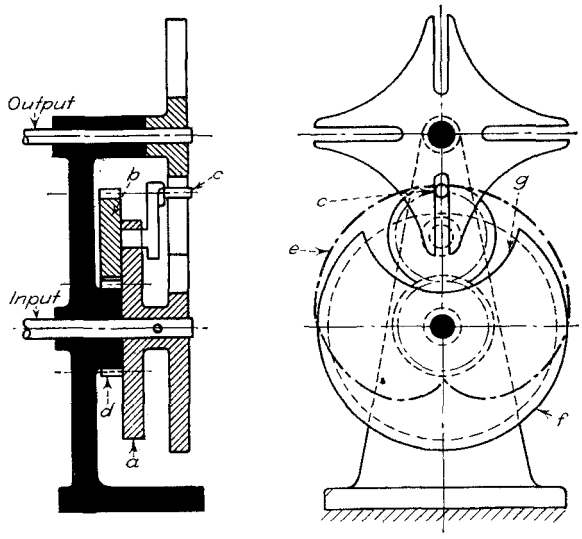


Fig. 2—(Above) This design incorporates a planet gear in the drive mechanism. The motion period of the output shaft is decreased and the maximum angular velocity is increased over that of an unmodified Geneva with the same number of slots. Crank wheel *a* drives the unit composed of plant gear *b* and driving roller *c*. The axis of the driving roller coincides with a point on the pitch circle of the planet gear; since the planet gear rolls around the fixed sun gear *d*, the axis of roller *c* describes a cardioid *e*. To prevent the roller from interfering with the locking disk *f*, the clearance arc *g* must be larger than required for unmodified Genevas.

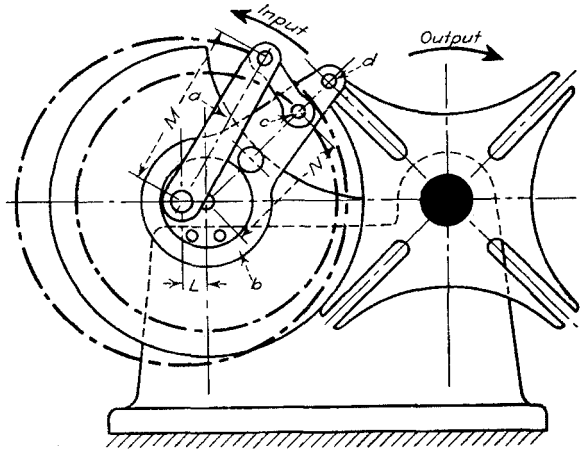


Fig. 3—A motion curve similar to that of Fig. 2 can be derived by driving a Geneva wheel by means of a two-crank linkage. Input crank *a* drives crank *b* through link *c*. The variable angular velocity of driving roller *d*, mounted on *b*, depends on the center distance *L*, and on the radii *M* and *N* of the crank arms. This velocity is about equivalent to what would be produced if the input shaft were driven by elliptical gears.

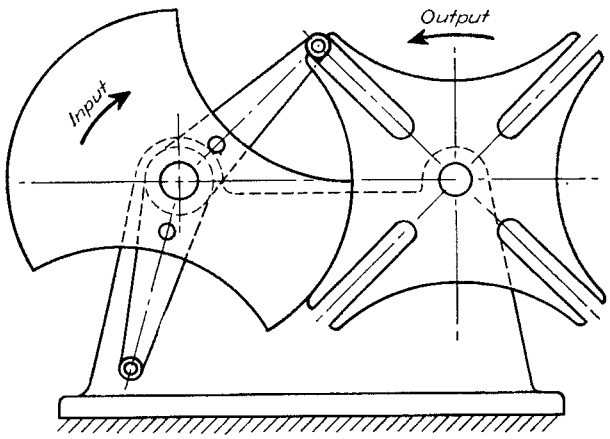


Fig. 5—(Below) In this intermittent drive, the two rollers drive the output shaft as well as lock it during dwell periods. For each revolution of the input shaft the output shaft has two motion periods. The output displacement ϕ is determined by the number of teeth; the driving angle, ψ, may be chosen within limits. Gear *a* is driven intermittently by two driving rollers mounted on input wheel *b*, which is bearing-mounted on frame *c*. During the dwell period the rollers circle around the top of a tooth. During the motion period, a roller's path *d* relative to the driven gear is a straight line inclined towards the output shaft. The tooth profile is a curve parallel to path *d*. The top land of a tooth becomes the arc of a circle of radius *R*, the arc approximating part of the path of a roller.

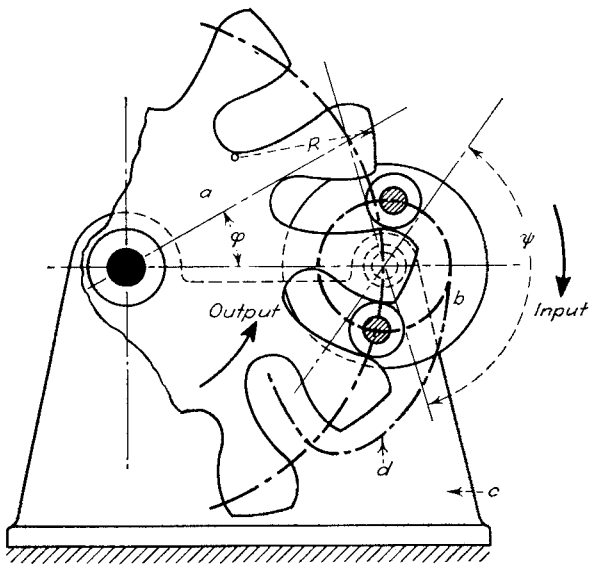


Fig. 4—(Left) The duration of the dwell periods is changed by arranging the driving rollers unsymmetrically around the input shaft. This does not affect the duration of the motion periods. If unequal motion periods are desired as well as unequal dwell periods, then the roller crank-arms must be unequal in length and the star must be suitably modified; such a mechanism is called an "irregular Geneva drive."

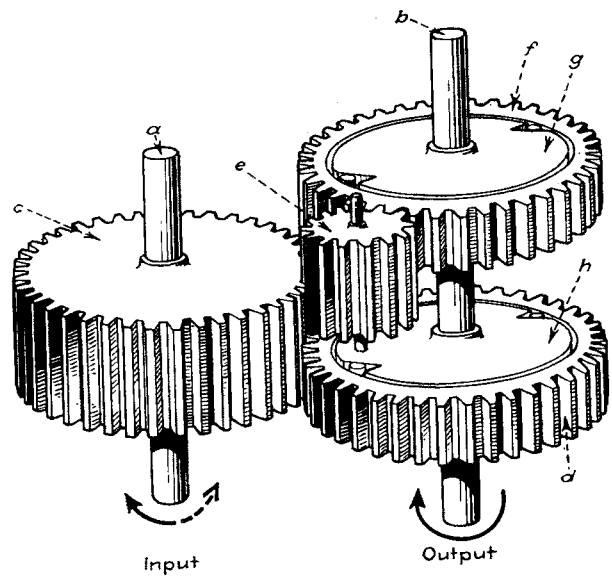


Fig. 6—This uni-directional drive was developed by the author and to his knowledge is novel. The output shaft rotates in the same direction at all times, without regard to the direction of the rotation of the input shaft; angular velocity of the output shaft is directly proportional to the angular velocity of the input shaft. Input shaft *a* carries spur gear *c*, which has approximately twice the face width of spur gears *f* and *d* mounted on output shaft *b*. Spur gear *c* meshes with idler *e* and with spur gear *d*. Idler *e* meshes with spur gears *c* and *f*. The output shaft *b* carries two free-wheel disks *g* and *h*, which are oriented uni-directionally.

When the input shaft rotates clockwise (bold arrow), spur gear *d* rotates counter-clockwise and idles around free-wheel disk *h*. At the same time idler *e*, which is also rotating counter-clockwise, causes spur gear *f* to turn clockwise and engage the rollers on free-wheel disk *g*; thus, shaft *b* is made to rotate clockwise. On the other hand, if the input shaft turns counter-clockwise (dotted arrow), then spur gear *f* will idle while spur gear *d* engages free-wheel disk *h*, again causing shaft *b* to rotate clockwise.

Epicyclic Gear Trains

M. F. Spotts

EPICYCLIC GEAR TRAIN shown in Fig. 1 has Arm A integral with the right hand shaft. Gears C and D are keyed to a short length of shaft which is mounted in a bearing in Arm A. Gear C meshes with the fixed internal gear B. Gear D meshes with internal gear E which is keyed to the left hand shaft.

To find the ratio of the speed of shaft E to the speed of shaft A proceed as follows. Let N_b be the number of teeth in gear B, N_c the number in gear C, and so on. Let arm A, which was originally in a vertical position, be given an angular displacement θ. In so doing gear C will traverse through arc ab on gear B. Arc bc of gear C must be equal to arc ab of gear B. Since angles are inversely proportional to radii, or to the number of teeth, gears C and D will have turned through angle $\theta N_b/N_c$.

While the foregoing was taking place gears D and E were rotating on each other through the equal arcs ed and ef. Gear E will have been turned in the reverse direction through angle

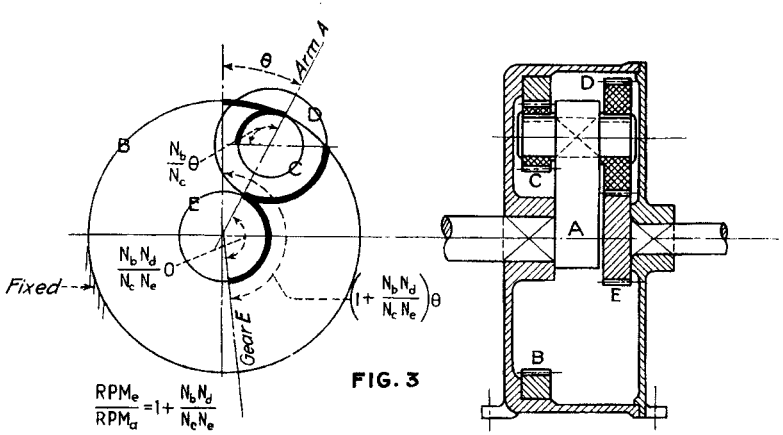
$$\theta N_b/N_c \times N_d/N_e$$

The net effect of these two operations is to move the point of gear E, which was originally vertical at g, over to location f. Gear E has thus been rotated through angle

$$(1 - N_bN_d/N_cN_e)\,\theta$$

This latter value when divided by θ, the angular movement of shaft A, gives the ratio of the rotations of shafts E and A respectively.

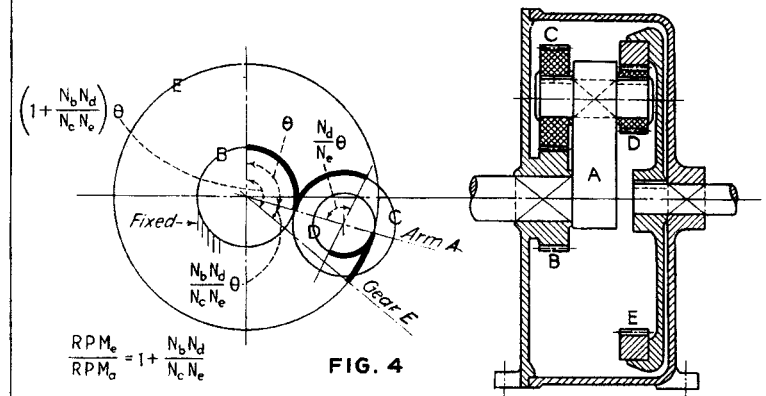
This method of analysis gives a graphical representation of the movement of all the parts. It may be easily applied to all types of epicyclic systems including those containing bevel gears. Additional examples are shown in Figs. 2 to 6 inclusive. Either of shafts A or E may be used as the driver.



$$\frac{RPM_e}{RPM_a}=1+\frac{N_b N_d}{N_c N_e}$$

FIG. 3

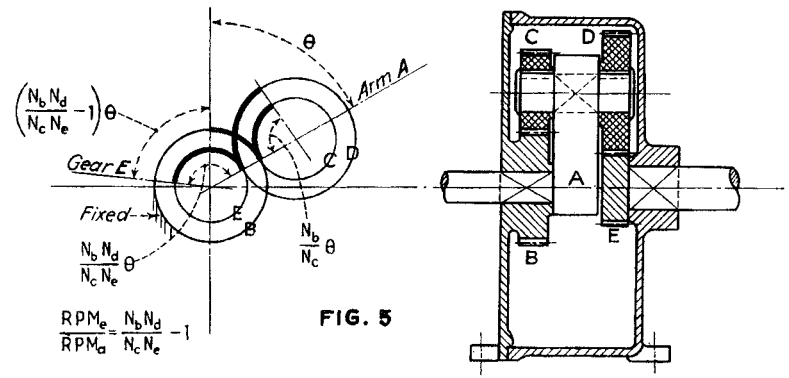
Drive and driven shafts rotate in **same** direction
Equation is valid for $N_c=N_d$, and for $N_c>N_d$



$$\frac{RPM_e}{RPM_a}=1+\frac{N_b N_d}{N_c N_e}$$

FIG. 4

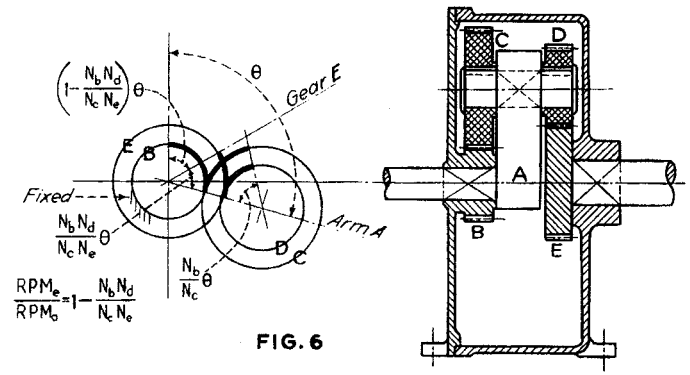
Drive and driven shafts rotate in **same** direction
Equation is valid for $N_c=N_d$, and for $N_c>N_d$



$$\frac{RPM_e}{RPM_a}=\frac{N_b N_d}{N_c N_e}-1$$

FIG. 5

Drive and driven shafts rotate in **opposite** directions



$$\frac{RPM_e}{RPM_a}=1-\frac{N_b N_d}{N_c N_e}$$

FIG. 6

Drive and driven shafts rotate in **same** direction

Cycloid Gear Mechanisms

Cycloidal motion is becoming popular for mechanisms in feeders and automatic machines. Here are arrangements, formulas, and layout methods.

Preben W. Jensen

THE appeal of cycloidal mechanisms is that they can easily be tailored to provide one of these three common motion requirements:

- **Intermittent motion**—with either short or long dwells
- **Rotary motion with progressive oscillation**—where the output undergoes a cycloidal motion during which the forward motion is greater than the return motion
- **Rotary-to-linear motion with a dwell period**

All the cycloidal mechanisms covered in this article are geared; this results in compact positive devices capable of operating at relatively high speeds with little backlash or "slop." The mechanisms can also be classified into three groups:

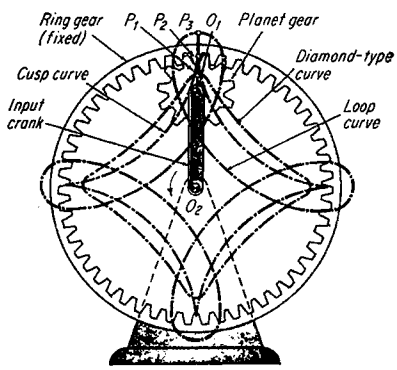
Hypocycloid—where the points tracing the cycloidal curves are located on an external gear rolling inside an internal ring gear. This ring gear is usually stationary and fixed to the frame.

Epicycloid—where the tracing points are on an external gear which rolls in another external (stationary) gear

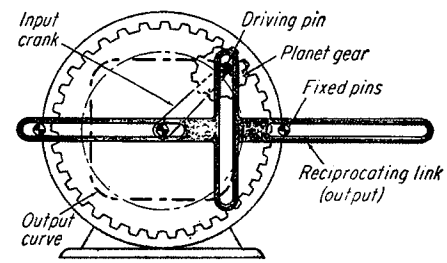
Pericycloid—where the tracing points are located on an internal gear which rolls on a stationary external gear.

HYPOCYCLOID MECHANISMS

1. Basic hypocycloid curves



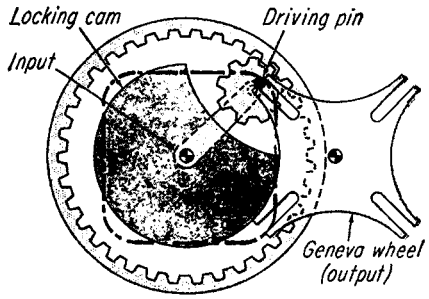
2. Double-dwell mechanism



Coupling the output pin to a slotted member produces a prolonged dwell in each of the extreme positions. This is another application of the diamond-type hypocycloidal curve.

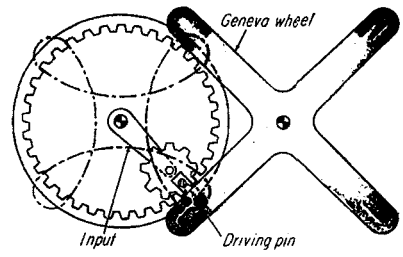
Input drives a planet in mesh with a stationary ring gear. Point P_1 on the planet gear describes a diamond-shape curve, point P_2 on the pitch line of the planet describes the familiar cusp curve, and point P_3, which is on an extension rod fixed to the planet gear, describes a loop-type curve. In one application, an end miller located at P_1 was employed in production for machining a diamond-shape profile.

3. Long-dwell geneva drive



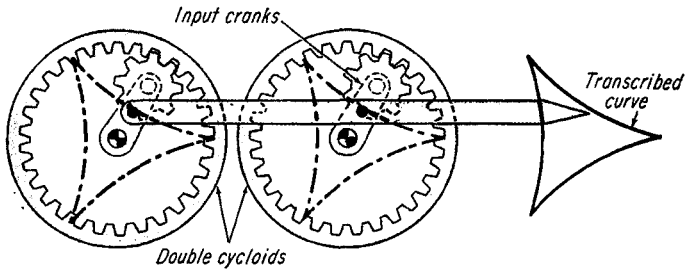
As with standard four-station genevas, each rotation of the input indexes the slotted geneva 90 deg. By employing a pin fastened to the planet gear to obtain a rectangular-shape cycloidal curve, a smoother indexing motion is obtained because the driving pin moves on a noncircular path.

4. Internal-geneva drive



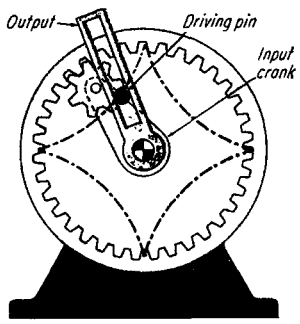
Loop-type curve permits driving pin to enter slot in a direction that is radially outward from the center, and then loop over to rapidly index the cross member. As with the previous geneva, the output rotates 90 deg, then goes into a long dwell period during each 270-deg rotation of the input.

5. Cycloidal parallelogram



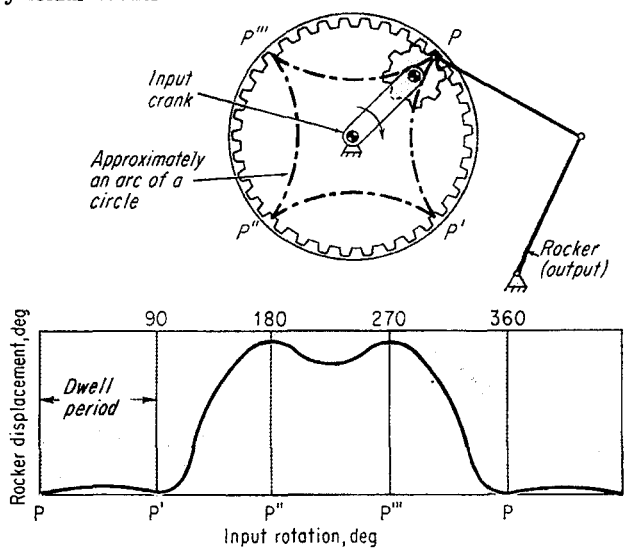
Two identical hypocycloid mechanisms guide the point of the bar along the triangularly shaped path. They are useful also in cases where there is limited space in the area where the curve must be described. Such double-cycloid mechanisms can be designed to produce other types of curves.

6. Short-dwell rotary



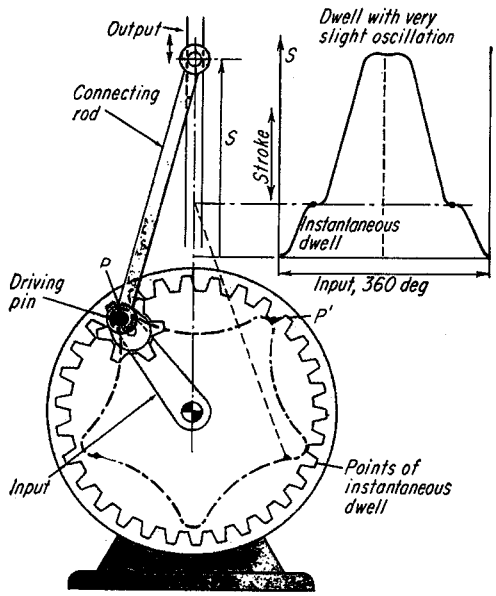
Here the pitch circle of the planet gear is exactly one-quarter that of the ring gear. A pin on the planet will cause the slotted output member to have four instantaneous dwells for each revolution of the input shaft.

7. Cycloidal rocker



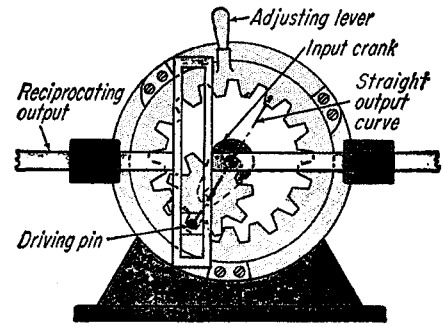
The curvature of the cusp is approximately that of an arc of a circle. Hence the rocker comes to a long dwell at the right extreme position while point P moves to P'. There is then a quick return from P' to P'', with a momentary dwell at the end of this phase. The rocker then undergoes a slight oscillation from point P'' to P''', as shown in the displacement diagram.

8. Cycloidal reciprocator



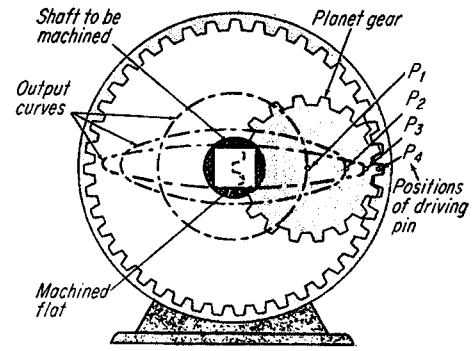
Portion of curve, *P-P'*, produces the long dwell (as in previous mechanism), but the five-lobe cycloidal curve avoids a marked oscillation at the end of the stroke. There are also two points of instantaneous dwell where the curve is perpendicular to the connecting rod.

9. Adjustable harmonic drive



By making the planet-gear half that of the internal gear, a straight-line output curve is produced by the driving pin which is fastened to the planet gear. The pin engages the slotted member to cause the output to reciprocate back and forth with harmonic (sinusoidal) motion. The position of the fixed ring gear can be changed by adjusting the lever, which in turn rotates the straight-line output-curve. When the curve is horizontal, the stroke is at a maximum; when the curve is vertical, the stroke is zero.

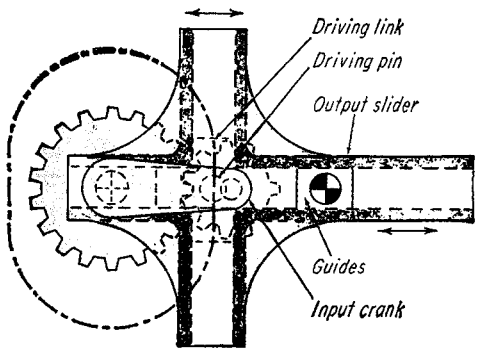
10. Elliptical-motion drive



By making the pitch diameter of the planet equal to half that of the ring gear, every point on the planet gear (such as points P_2 and P_3) will describe elliptical curves which get flatter as the points are selected closer to the pitch circle. Point P_1, at the center of the planet, describes a circle; point P_4 at the pitch circle describes a straight line. When a cutting tool is placed at P_3, it will cut almost-flat sections from round stock, as when machining a bolt. The other two sides of the bolt can be cut by rotating the bolt, or the cutting device, 90 deg. (Reference: H. Zeile, *Unrund- und Mehrkantdrehen,* VDI-Berichte, Nr. 77,1965.)

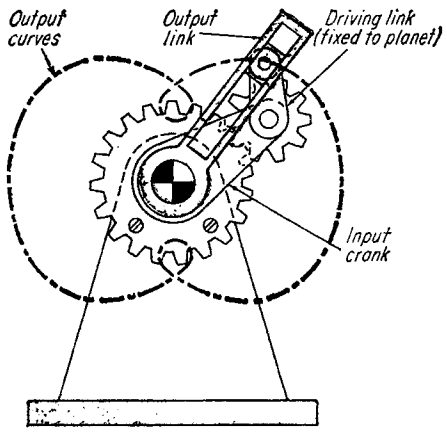
EPICYCLOID MECHANISMS

11. Epicycloid reciprocator



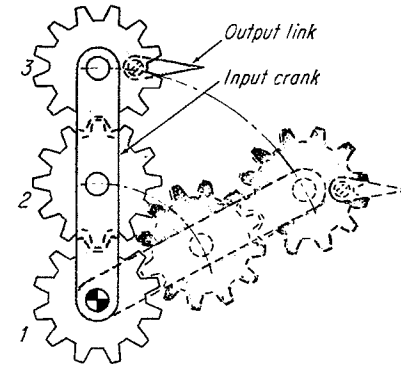
Here the sun gear is fixed and the planet gear driven around it by means of the input link. There is no internal ring gear as with the hypocycloid mechanisms. Driving pin *P* on the planet describes the curve shown which contains two almost-flat portions. By having the pin ride in the slotted yoke, a short dwell is produced at both the extreme positions of the output member. The horizontal slots in the yoke ride the end-guides, as shown.

12. Progressive oscillating drive



By fixing a crank to the planet gear, a point P can be made to describe the double loop curve illustrated. The slotted output crank oscillates briefly at the vertical portions.

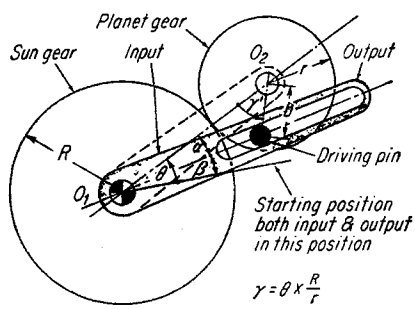
13. Parallel-guidance mechanisms



The input crank contains two planet gears. The center sun-gear is fixed as in the previous epicycloid mechanisms. By making the three gears equal in diameter and having gear *2* serve as an idler, any member fixed to gear *3* will remain parallel to its previous positions throughout the rotation of the input ring crank.

MOTION EQUATIONS

14. Equations for epicycloid drives



The equations for angular displacement, velocity and acceleration for basic epicyclic drive are given below. (Reference: Schmidt, E. H., "Cycloidal Cranks," *Transactions of the 5th Conference on Mechanisms,* 1958, pp 164-180):

Angular displacement

$$\tan \beta = \frac{(R + r) \sin \theta - b \sin (\theta + \gamma)}{(R + r) \cos \theta - b \cos (\theta + \gamma)} \qquad (1)$$

Angular velocity

$$V = \omega \frac{1 + \frac{b^2}{r(R + r)} - \left(\frac{2r + R}{r}\right)\left(\frac{b}{R + r}\right)\left(\cos \frac{R}{r}\theta\right)}{1 + \left(\frac{b}{R + r}\right)^2 - \left(\frac{2b}{R + r}\right)\left(\cos \frac{R}{r}\theta\right)} \qquad (2)$$

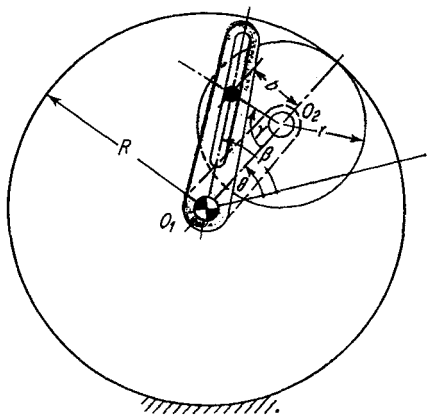
Angular acceleration

$$A = \omega^2 \frac{\left(1 - \frac{b^2}{(R + r)^2}\right)\left(\frac{R^2}{r^2}\right)\left(\frac{b}{R + r}\right)\left(\sin \frac{R}{r}\theta\right)}{\left[1 + \frac{b^2}{(R + r)^2} - \left(\frac{2b}{R + r}\right)\left(\cos \frac{R}{r}\theta\right)\right]^2} \qquad (3)$$

Symbols

A = angular acceleration of output, deg/sec^2
b = radius of driving pin from center of planet gear
r = pitch radius of planet gear
R = pitch radius of fixed sun gear
V = angular velocity of output, deg/sec
β = angular displacement of output, deg
$\gamma = \theta R/r$
θ = input displacement, deg
ω = angular velocity of input, deg/sec

15. Equations for hypocycloid drives



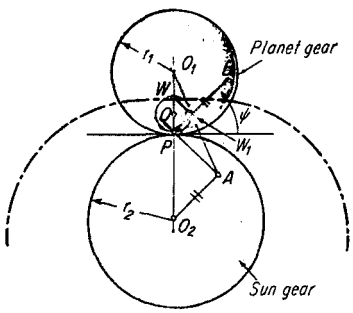
$$\tan\beta = \frac{\sin\theta - \left(\frac{b}{R-r}\right)\left(\sin\frac{R-r}{r}\theta\right)}{\cos\theta + \left(\frac{b}{R-r}\right)\left(\cos\frac{R-r}{r}\theta\right)} \quad (4)$$

$$V = \omega\,\frac{1-\left(\frac{R-r}{r}\right)\left(\frac{b^2}{(R+r)^2}\right)+\left(\frac{2r-R}{r}\right)\left(\cos\frac{R}{r}\theta\right)}{1+\frac{b^2}{(R+r)^2}+\left(\frac{2b}{R+r}\right)\left(\cos\frac{R}{r}\theta\right)} \quad (5)$$

$$A = \omega^2\,\frac{\left(1-\frac{b^2}{(R+r)^2}\right)\left(\frac{b}{R+r}\right)\left(\frac{R^2}{r^2}\right)\left(\sin\frac{R}{r}\theta\right)}{\left[1+\frac{b^2}{(R+r)^2}+\left(\frac{2b}{R+r}\right)\left(\cos\frac{R}{r}\theta\right)\right]^2} \quad (6)$$

DESCRIBING APPROXIMATE STRAIGHT LINES

16. Gear rolling on a gear—flatten curves

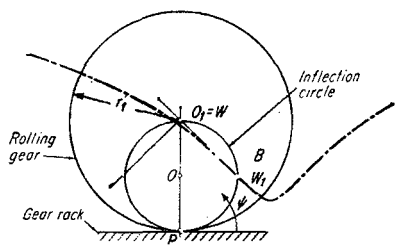


It is frequently desirable to find points on the planet gear that will describe approximately straight lines for portions of the output curve. Such points will yield dwell mechanisms, as shown in Fig 2 and 11. Construction is as follows (shown at left):

1. Draw an arbitrary line *PB*.
2. Draw its parallel O_2A.
3. Draw its perpendicular *PA* at *P*. Locate point *A*.
4. Draw O_1A. Locate W_1.
5. Draw perpendicular to PW_1 at W_1 to locate *W*.
6. Draw a circle with *PW* as the diameter.

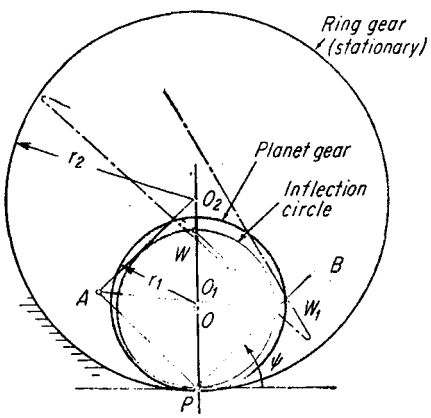
All points on this circle describe curves with portions that are approximately straight. This circle is also called the inflection circle because all points describe curves which have a point of inflection at the position illustrated. (Shown is the curve passing through point *W*.)

17. Gear rolling on a rack—vee curves



This is a special case. Draw a circle with a diameter half that of the gear (diameter O_1P). This is the inflection circle. Any point, such as point W_1, will describe a curve that is almost straight in the vicinity selected. Tangents to the curves will always pass through the center of the gear, O_1 (as shown).

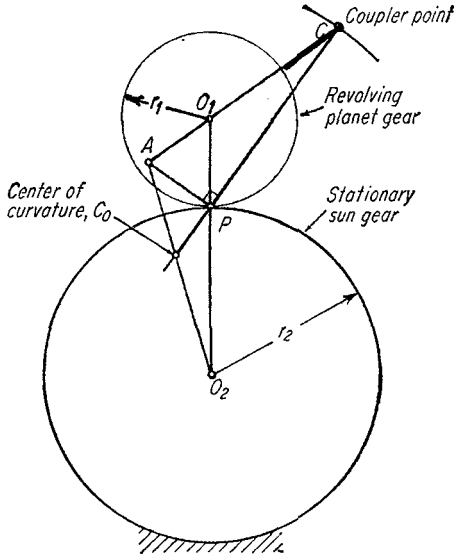
18. Gear rolling inside a gear—zig-zag



To find the inflection circle for a gear rolling inside a gear:

1. Draw arbitrary line *PB* from the contact point *P*.
2. Draw its parallel O_2A, and its perpendicular, *PA*. Locate *A*.
3. Draw line AO_1 through the center of the rolling gear. Locate W_1.
4. Draw a perpendicular through W_1. Obtain *W*. Line *WP* is the diameter of the inflection circle. Point W_1, which is an arbitrary point on the circle, will trace a curve of repeated almost-straight lines, as shown.

19. Center of curvature—gear rolling on gear

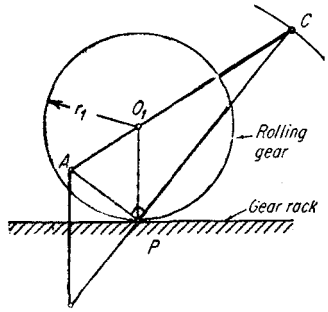


By locating the centers of curvature at various points, one can then determine the proper length of the rocking or reciprocating arm to provide long dwells (as required for the mechanisms in Fig 7 and 8), or proper entry conditions (as for the drive pin in the mechanism in Fig 3).

In the case of a gear with an extended point, point C, rolling on another gear, the graphical method for locating the center of curvature is given by these steps:

1. Draw a line through points C and P.
2. Draw a line through points C and O_1.
3. Draw a perpendicular to CP at P. This locates point A.
4. Draw line AO_2, to locate C_o, the center of curvature.

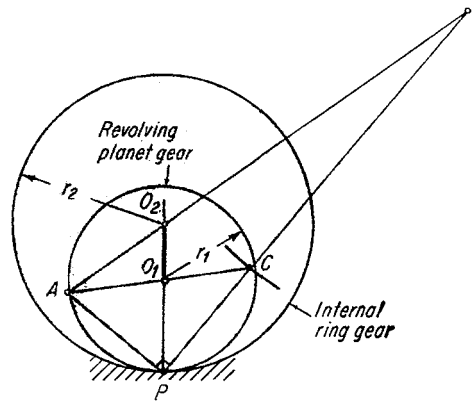
20. Center of curvature—gear rolling on a rack



Construction is similar to that of the previous case.

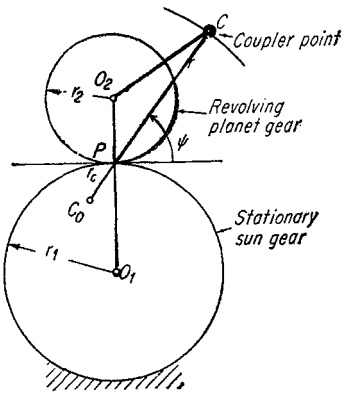
1. Draw an extension of line CP.
2. Draw a perpendicular at P to locate A.
3. Draw a perpendicular from A to the straight suface to locate C_o.

21. Center of curvature—gear rolling inside a gear



1. Draw extensions of CP and CO_1.
2. Draw a perpendicular of PC at P to locate A.
3. Draw AO_2 to locate C_o.

22. Analytical solutions



The centure of curvature of a gear rolling on a external gear can be computed directly from the Euler-Savary equation:

$$\left(\frac{1}{r} - \frac{1}{r_c}\right)\sin\psi = \text{constant} \quad (7)$$

where angle ψ and r locate the position of C.

By applying this equation twice, specifically to point O_1 and O_2 which have their own centers of rotation, the following equation is obtained:

$$\left(\frac{1}{r_2} + \frac{1}{r_1}\right)\sin 90° = \left(\frac{1}{r} + \frac{1}{r_c}\right)\sin\psi$$

or

$$\frac{1}{r_2} + \frac{1}{r_1} = \left(\frac{1}{r} + \frac{1}{r_c}\right)\sin\psi$$

This is the final design equation. All factors except r_c are known: hence solving for r_c leads to the location of C_o.

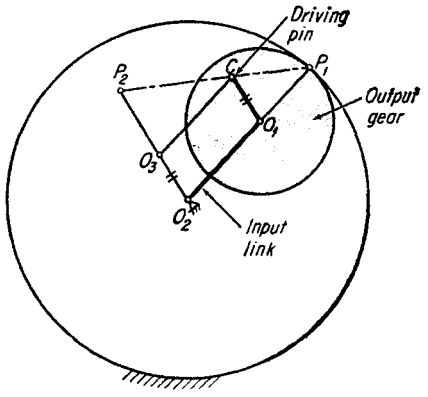
For a gear rolling inside an internal gear, the Euler-Savary equation is

$$\left(\frac{1}{r} + \frac{1}{r_c}\right)\sin\psi = \text{constant}$$

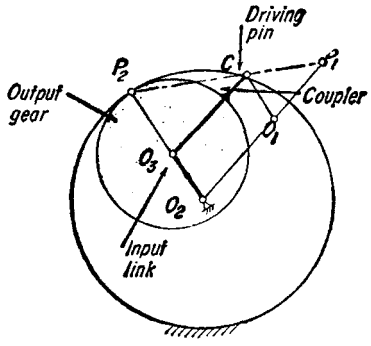
which leads to

$$\frac{1}{r_2} - \frac{1}{r_1} = \left(\frac{1}{r} - \frac{1}{r_c}\right)\sin\psi$$

23. Hypocycloid substitute



Original mechanism

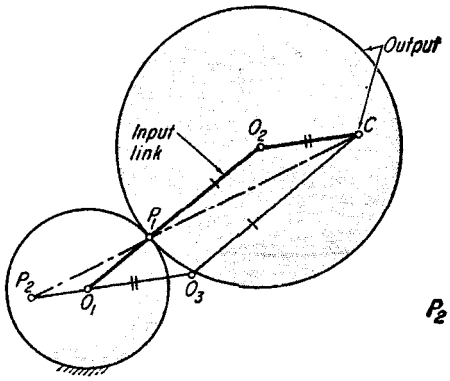


Substitute

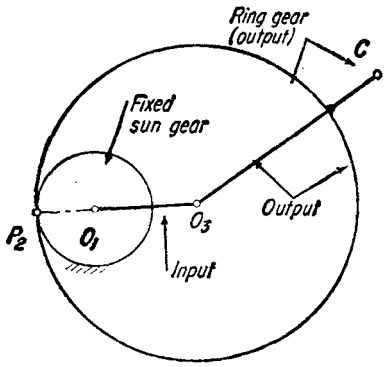
It is not always realized that cycloid mechanisms can frequently be replaced by other cycloids that produce the same motion and yet are more compact.

The mechanism (right) is a typical hypocycloid. Gear *1* rolls inside gear *2* while point *C* describes a hypocycloid curve. To find the substitute mechanism, draw parallels O_3O_2 and O_3C to locate point P_2. Then select O_2P_2 as the new radius of the large (internal) gear. Line P_2O_3 becomes the radius of the small gear. Point *C* has the same relative position and can be obtained by completing the triangles. The new mechanism is about two-thirds the size of the original.

24. Epicycloid substitute



Original mechanism

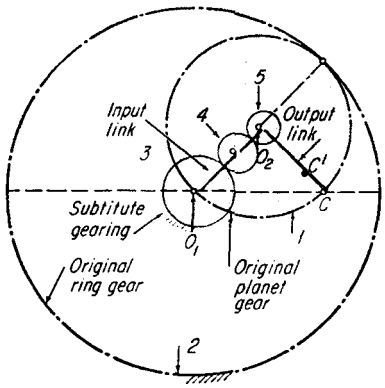


Substitute

The equivalent mechanisms of epicycloids are pericycloids in which the planetary gear is stationary and the output is taken from the ring gear. Such arrangements usually lead to a more-compact design.

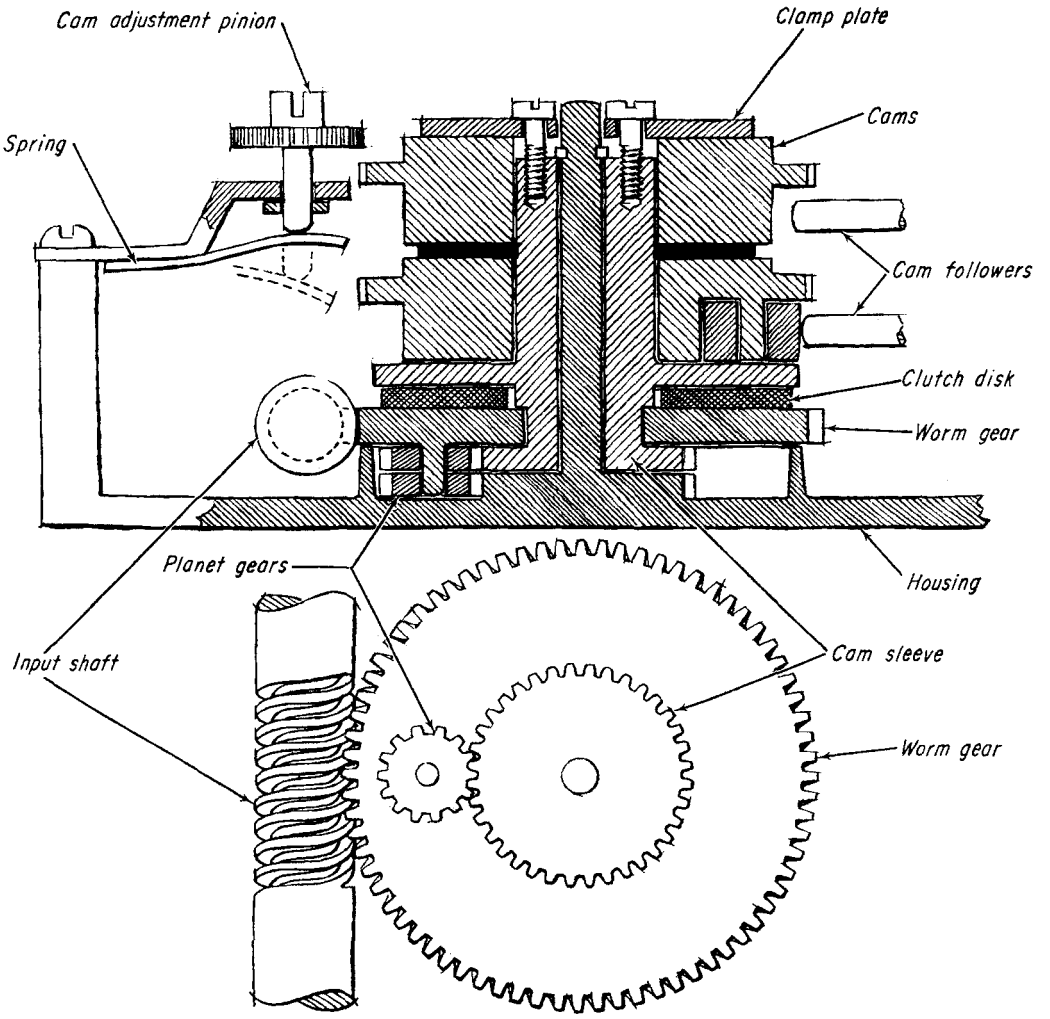
In the above mechanism, point *C* traces an epicycloidal curve. Draw the proper parallels to find P_2, then use P_2O_3 to construct the compact substitute mechanism shown at right of original.

25. Multigear substitute



This is another way of producing a compact substitute for a hypocycloid mechanism. The original mechanism is shown in dashed lines—gear *1* rolls inside gear *2* and point *C* describes the curve. The three external gears (gears *3*, *4*, and *5*) replace gears *1* and *2* with a remarkable savings in space. The only criterion is that gear *5* must be one-half the size of gear *3*: gear *4* is only an idler. The new mechanism thus has been reduced to approximately one-half that of the original in size.

Limit-Switch Backlash



SWITCH-ACTUATING CAMS are driven by double-reduction gearing. The first pass is the input worm and its worm gear. The second reduction consists of a planetary system with two keyed planets pivoted on the worm gear. The two planets do not have the same number of teeth. When the worm gear is rotated the planet gears move around a sun gear cast into the base of the housing. The upper planet meshes with gear teeth on the sleeve. The cams clamped to the sleeve actuate the switches. The ratio of the planetary reduction can be altered by changing the planets.

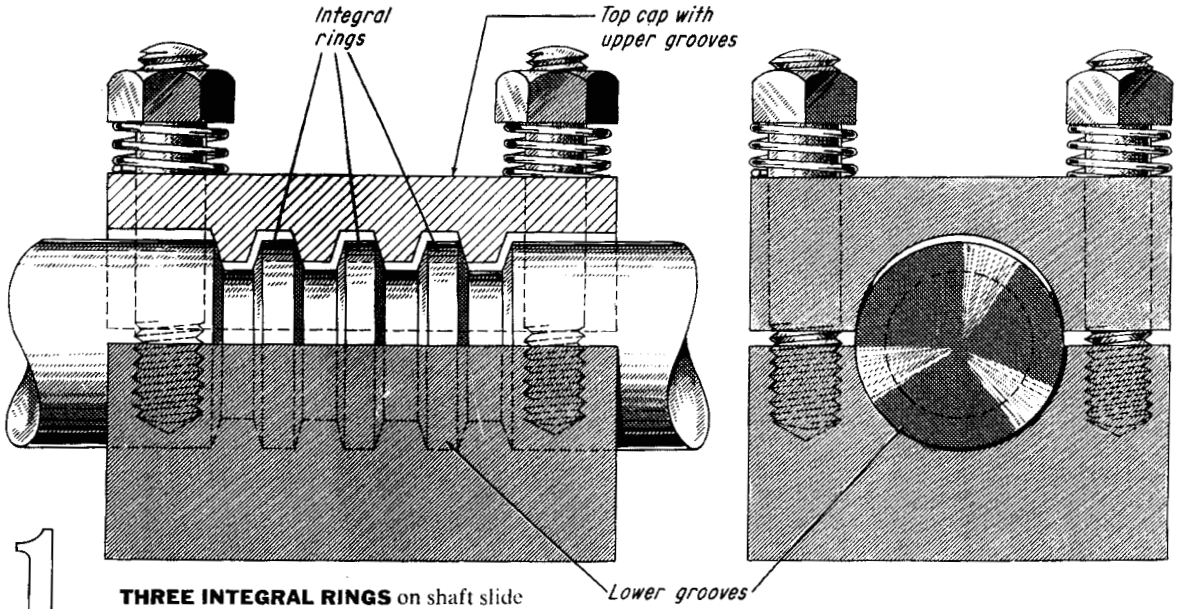
A friction clutch between the sleeve flange and the worm gear makes the switch exceedingly sensitive to reversals at the input worm. When a switch is actuated to reverse input direction, the cams are driven directly by the input worm and gear through the friction clutch until the backlash has been taken up. At this point the clutch begins to slip. The immediate reversal of the cams resets switches in ⅓ to 1 revolution depending on the worm-gear ratio.

In some of the reduction ratios available a deliberate mismatch is employed in planetary gear sizes. This intentional mismatch creates no problems at the pitch velocities produced, since the 3500 rpm maximum at the input shaft is reduced by the step-down of the worm and worm gear. The low torque requirements of the switch-operating cams eliminate any overstressing due to mismatch. The increased backlash obtained by the mismatch is desirable in the higher reduction ratios to allow the friction clutch to reset the switches before the backlash is taken up. This permits switch reset in less than 1 rpm despite the higher input gear reductions of as much as 1280:1.

4 Ways to Eliminate Backlash

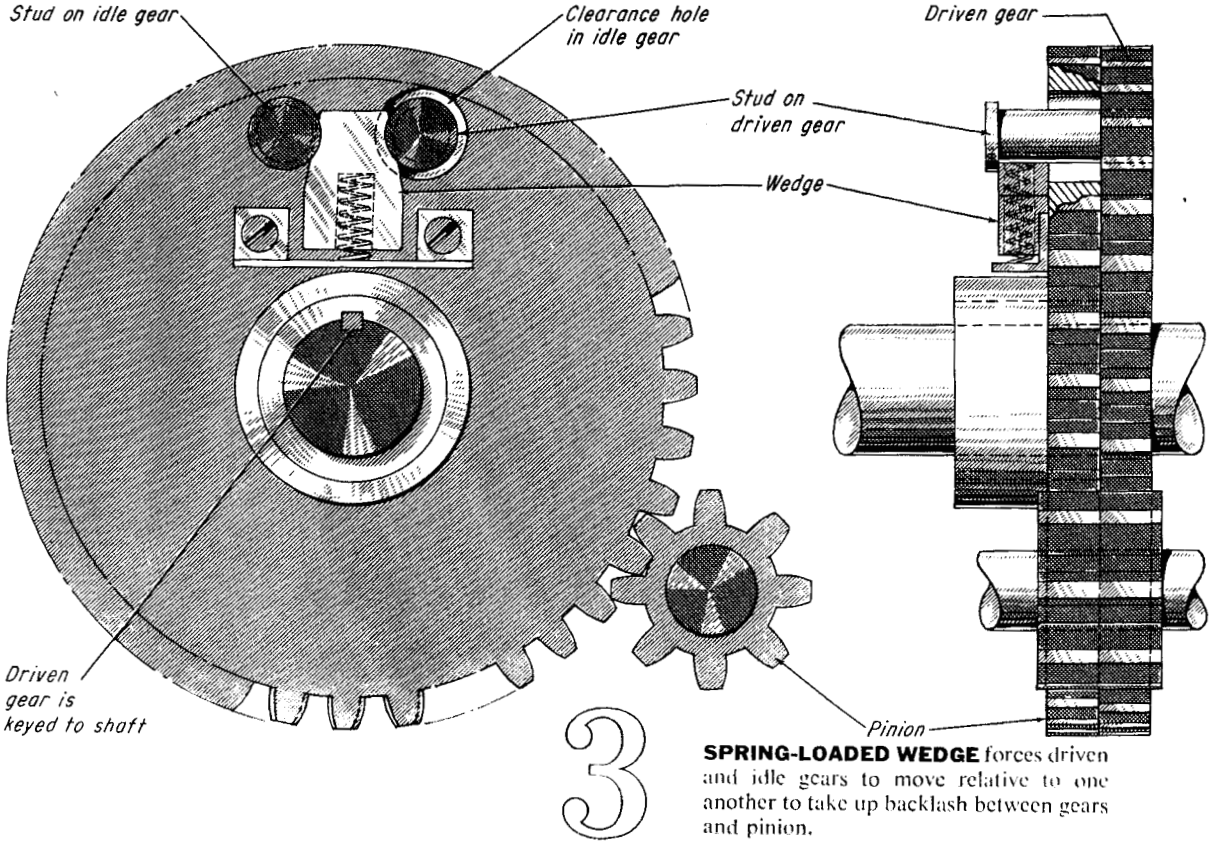
Wedges take up freedom in threads and gears, hold shaft snug against bearing.

L. Kasper



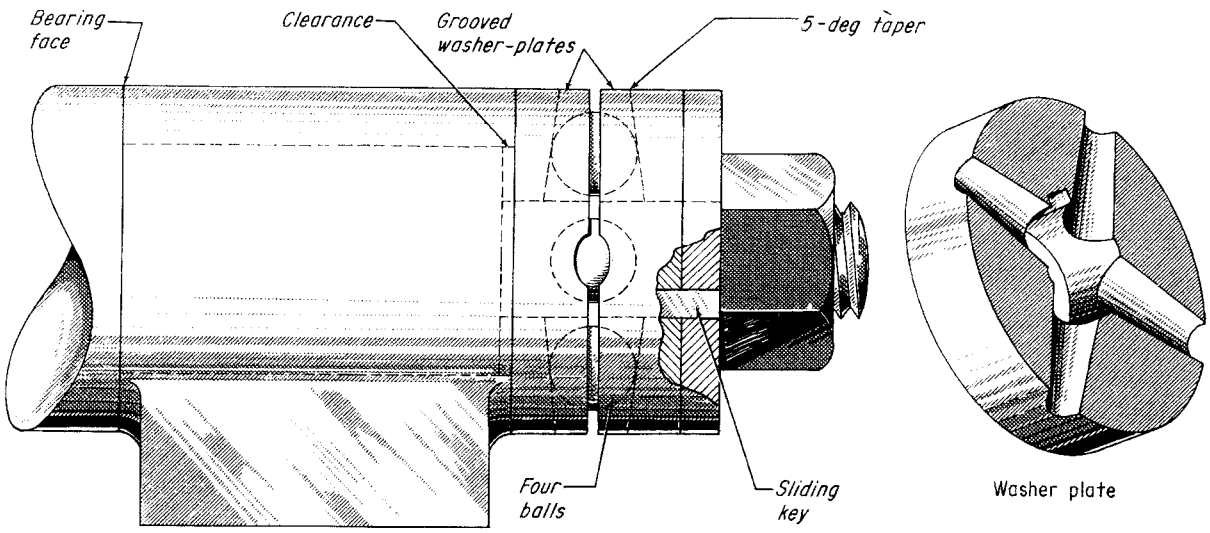
1

THREE INTEGRAL RINGS on shaft slide in grooves to prevent axial movement of shaft. Grooves in cap are offset axially.

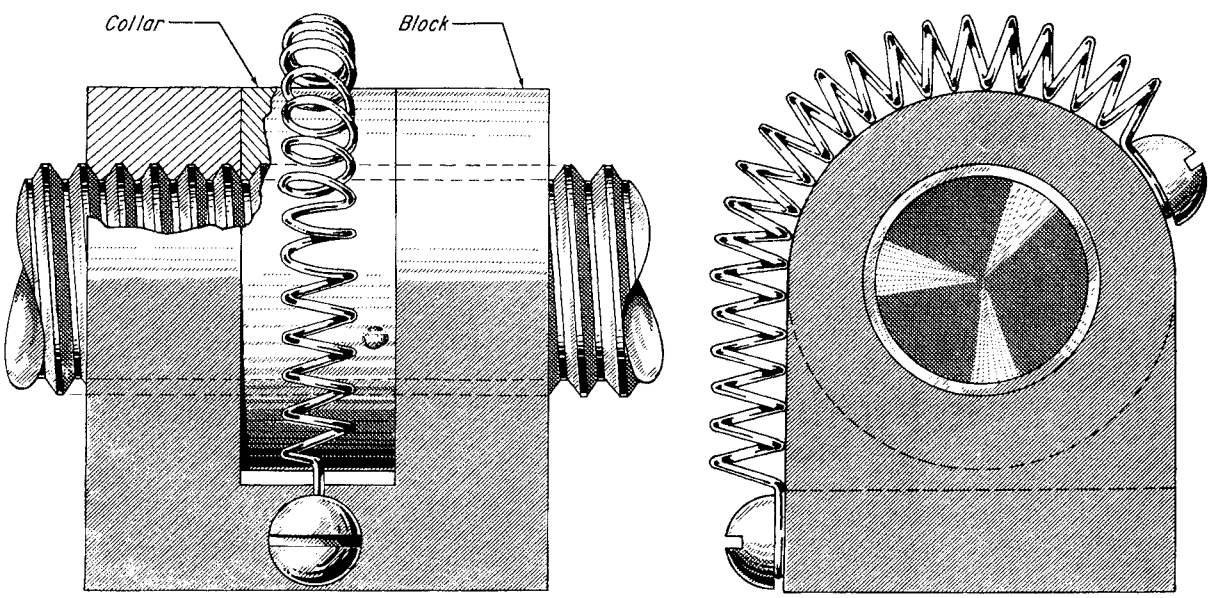


3

SPRING-LOADED WEDGE forces driven and idle gears to move relative to one another to take up backlash between gears and pinion.



2 **CENTRIFUGAL FORCE** causes balls to exert force on grooved washer-plates when shaft rotates, pulling it against bearing face.

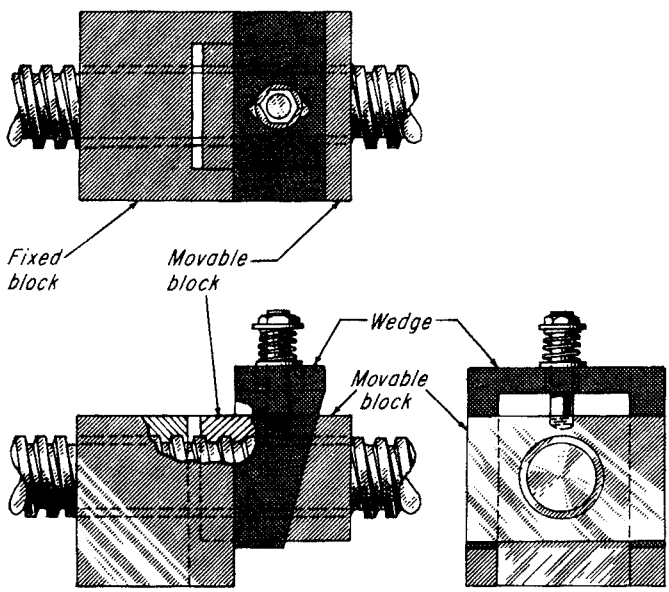


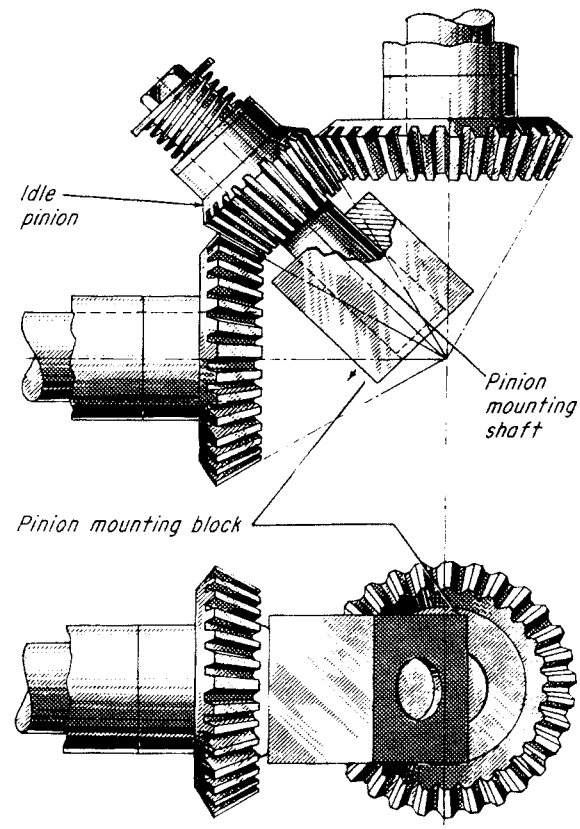
4 **COLLAR AND BLOCK** have continuous V-thread. When wear takes place in lead screw, the collar always maintains pressure on threads.

4 More Ways to Prevent Backlash

Springs combine with wedging action to ensure that threads, gears and toggles respond smoothly.

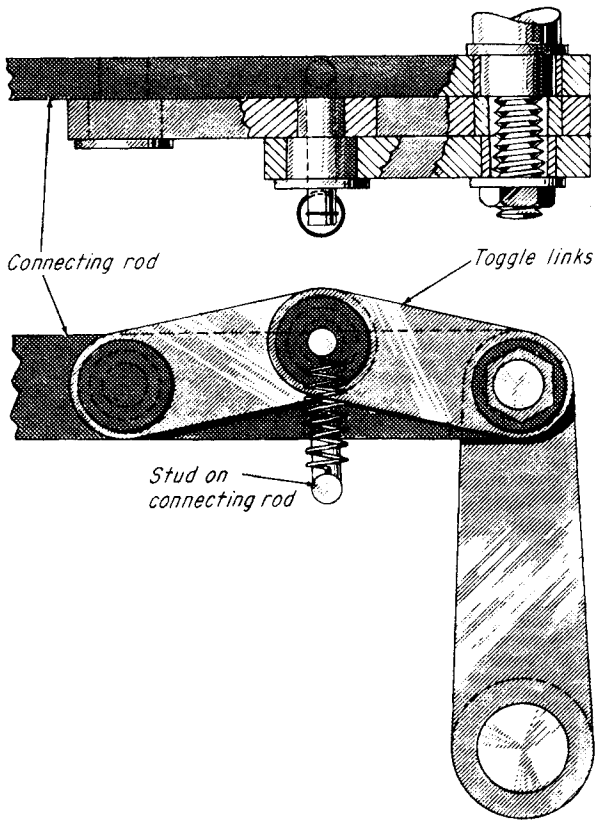
L. Kasper



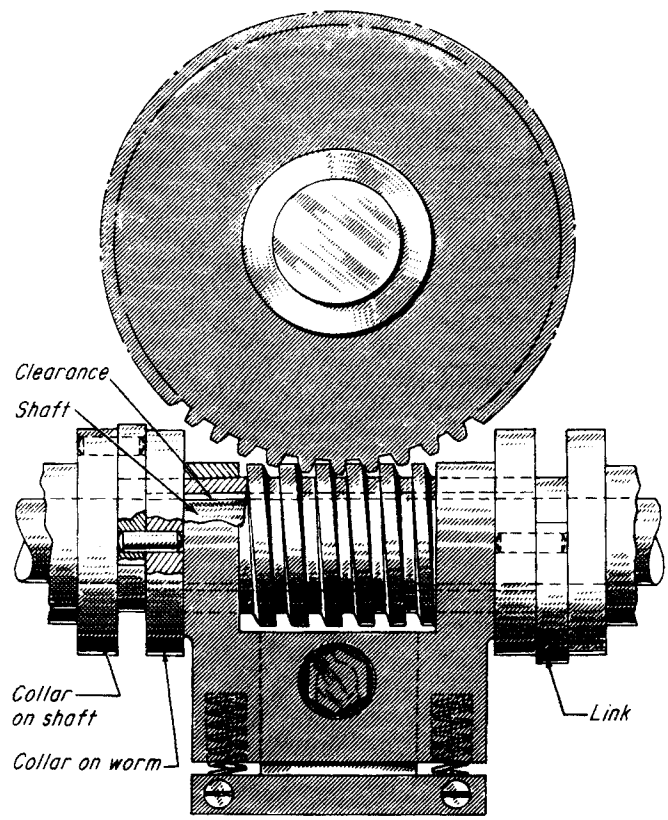


2 **SPRING-LOADED PINION** is mounted on a shaft located so that the spring forces pinion teeth into gear teeth to take up lost motion or backlash.

1 **MOVABLE BLOCK** is forced away from fixed block by spring-loaded wedge. Pressure is applied to both sides of lead screw, thus ensuring snug fit.



3 **TOGGLE LINKS** are spring-loaded and approach alignment to take up lost motion as wear in the joint takes place. Smooth response is thus gained.

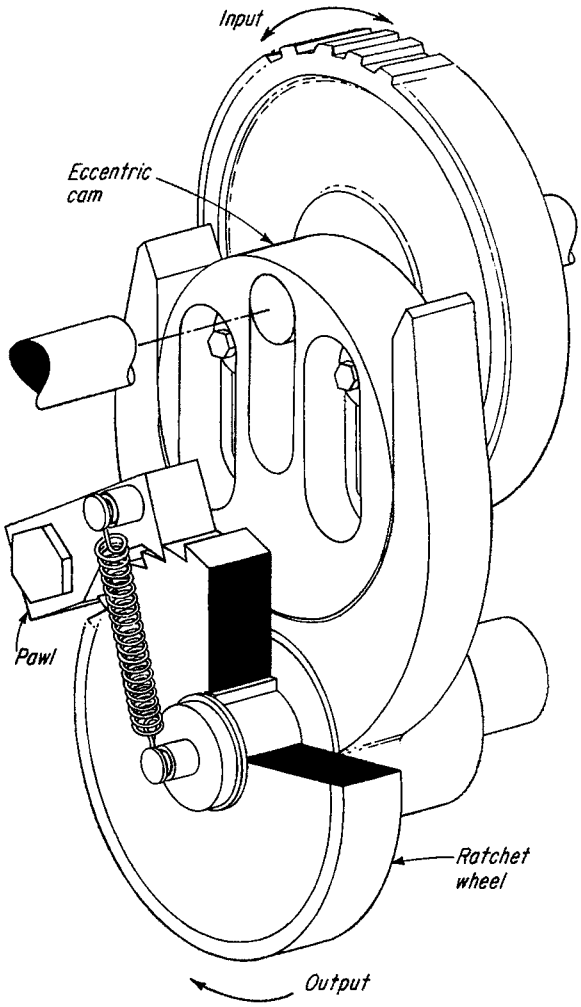


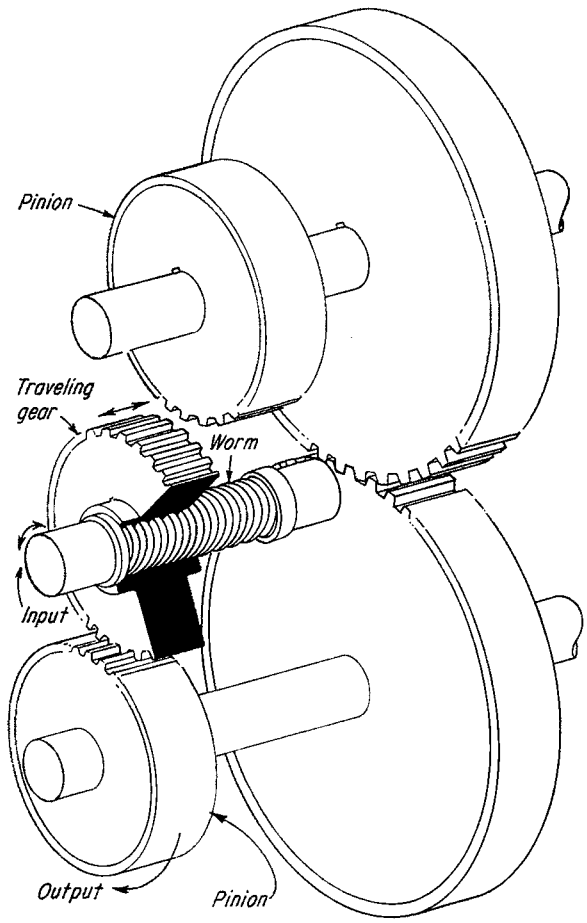
4 **HOLLOW WORM** has clearance for shaft, which drives worm through pinned collars and links. As wear occurs, springs move worm into teeth.

1-Way Output from Speed Reducers

When input reverses, these 5 slow-down mechanisms continue supplying a non-reversing rotation.

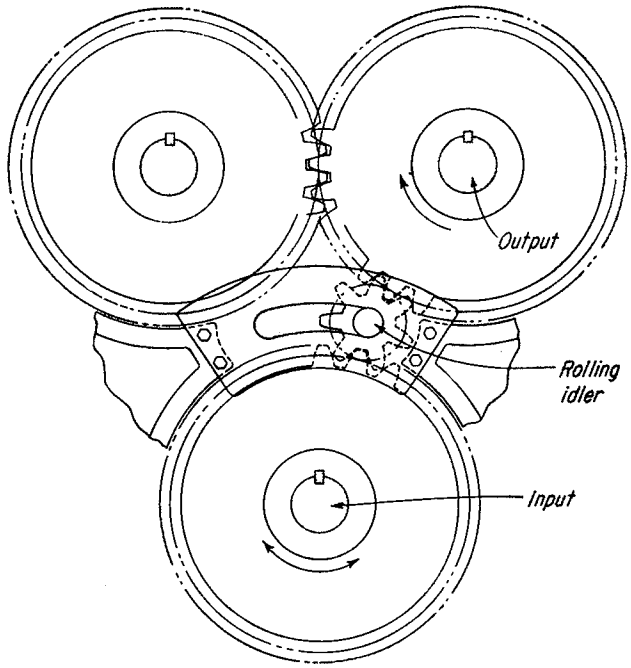
Louis Slegel



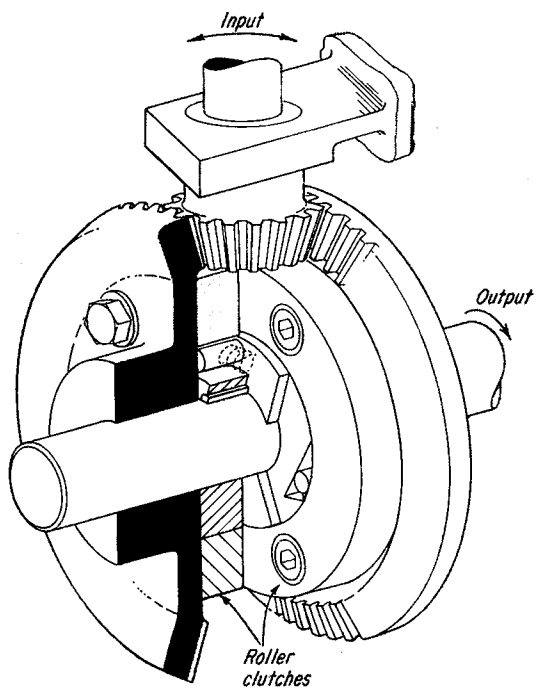


1 ECCENTRIC CAM adjusts over a range of high-reduction ratios, but unbalance limits it to low speeds. When direction of input changes, there is no lag in output rotation. Output shaft moves in steps because of ratchet drive through pawl which is attached to U-follower.

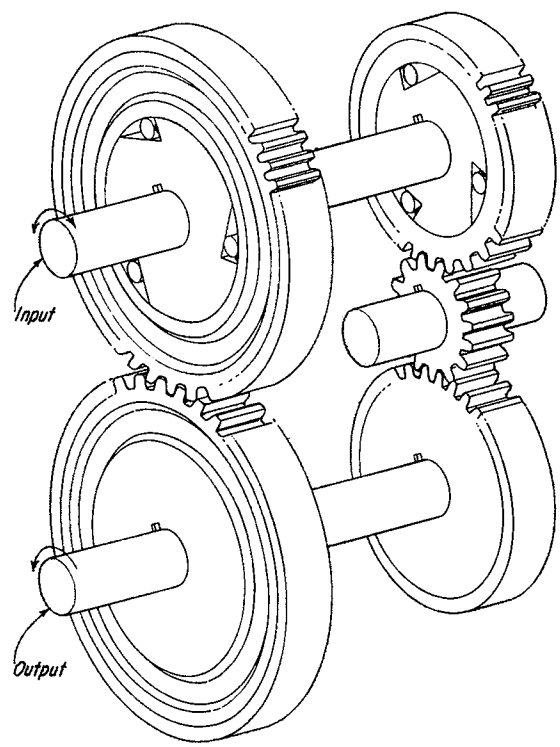
2 TRAVELING GEAR moves along worm and transfers drive to other pinion when input rotation changes direction. To ease engagement, gear teeth are tapered at ends. Output rotation is smooth, but there is a lag after direction changes as gear shifts. Gear cannot be wider than axial offset between pinions, or there will be interference.



3 ROLLING IDLER also gives smooth output and slight lag after input direction changes. Small drag on idler is necessary, so that it will transfer into engagement with other gear and not sit spinning in between.



4 TWO BEVEL GEARS drive through roller clutches. One clutch catches in one direction; the other catches in the opposite direction. There is negligible interruption of smooth output rotation when input direction changes.

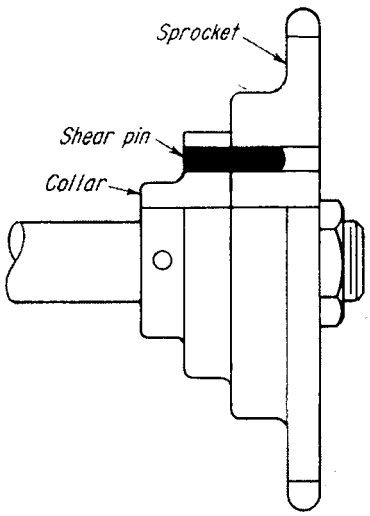


5 ROLLER CLUTCHES are on input gears in this drive, again giving smooth output speed and little output lag as input direction changes.

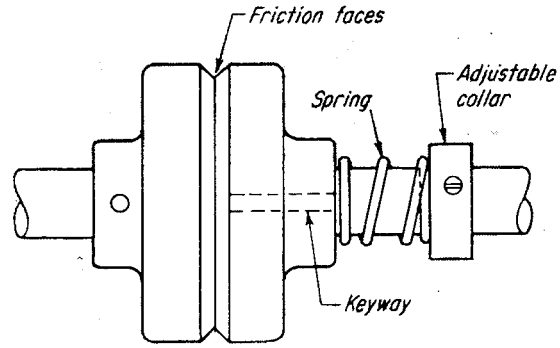
6 Ways to Prevent Overloading

These "safety valves" give way if machinery jams, thus preventing serious damage.

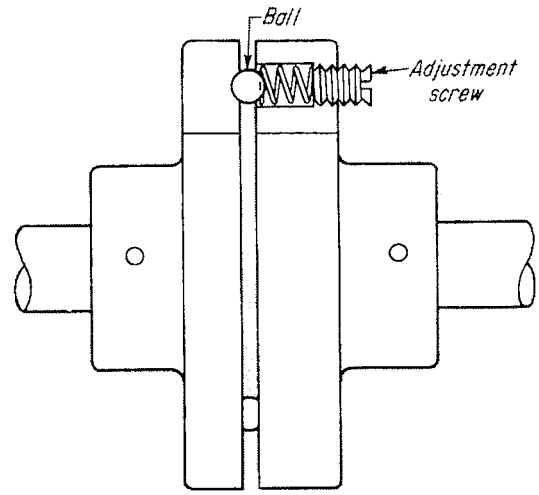
Peter C. Noy



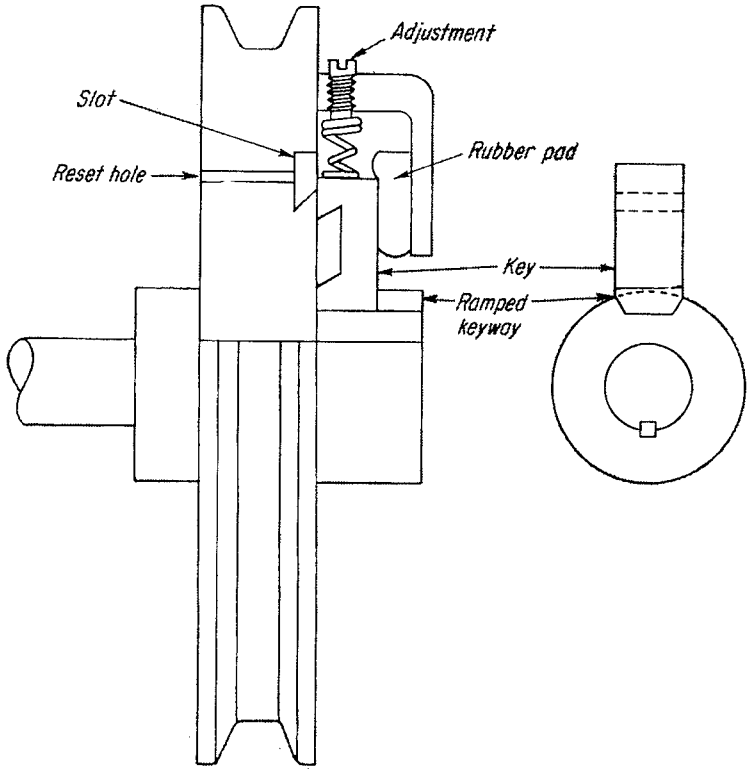
1
SHEAR PIN is simple to design and reliable in service. However, after an overload, replacing the pin takes a relatively long time; and new pins aren't always available.



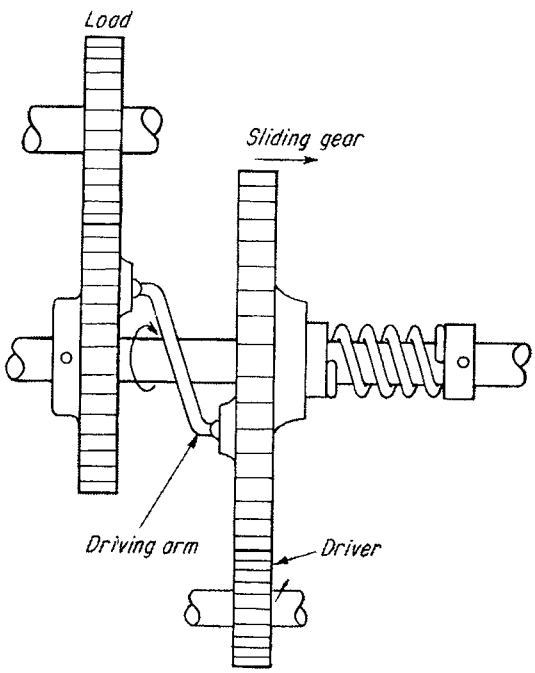
2
FRICTION CLUTCH. Adjustable spring tension that holds the two friction surfaces together sets overload limit. As soon as overload is removed the clutch reengages. One drawback is that a slipping clutch can destroy itself if unnoticed.

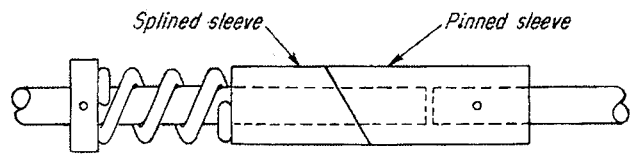


3
MECHANICAL KEYS. Spring holds ball in dimple in opposite face until overload forces the ball out. Once slip begins, wear is rapid, so device is poor when overload is common.



4
RETRACTING KEY. Ramped sides of keyway force key outward against adjustabe spring. As key moves outward, a rubber pad—or another spring—forces the key into a slot in the sheave. This holds the key out of engagement and prevents wear. To reset, push key out of slot by using hole in sheave.





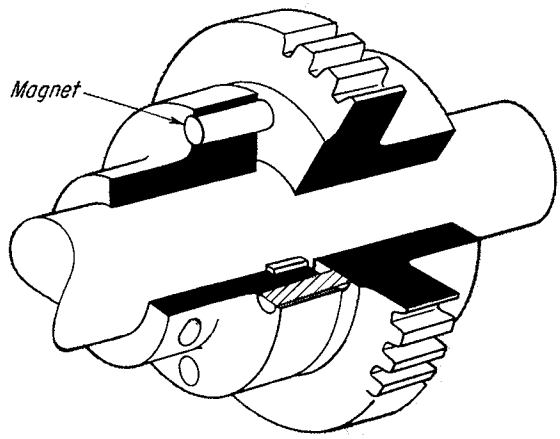
5
ANGLE-CUT CYLINDER. With just one tooth, this is a simplified version of the jaw clutch. Spring tension sets load limit.

6
DISENGAGING GEARS. Axial forces of spring and driving arm balance. Overload overcomes spring force to slide gears out of engagement. Gears can strip once overloading is removed, unless a stop holds gears out of engagement.

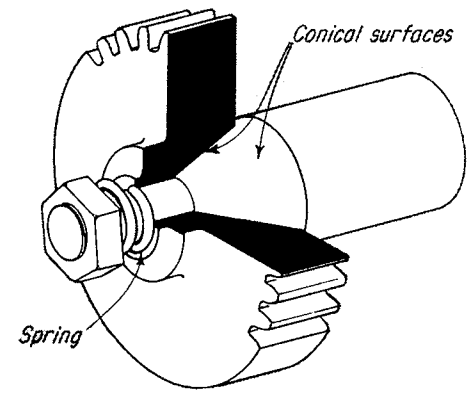
Torque-Limiters Protect Light-Duty Drives

In such drives the light parts break easily when overloaded. These eight devices disconnect them from dangerous torque surges.

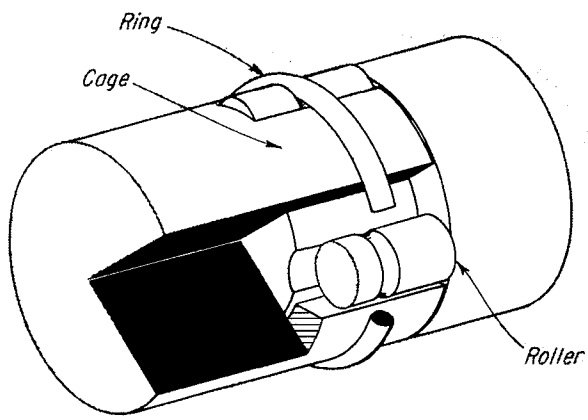
L. Kasper



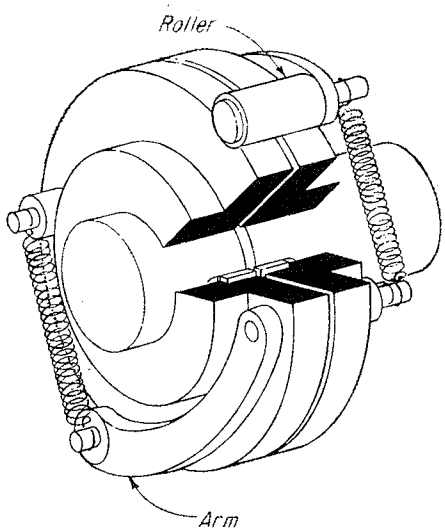
1
MAGNETS transmit torque according to their number and size. In-place control is limited to lowering torque capacity by removing magnets.



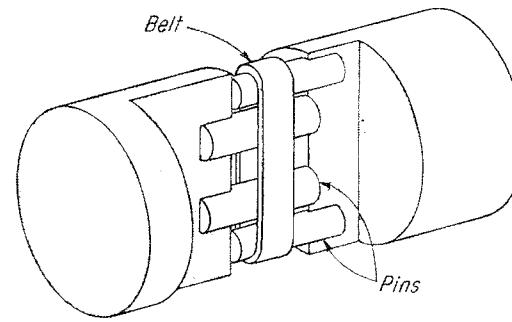
2
CONE CLUTCH is formed by mating taper on shaft to beveled hole through gear. Tightening down on nut increases torque capacity.



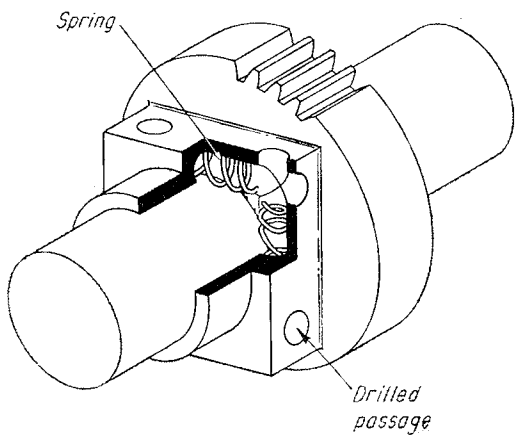
3
RING fights natural tendency of rollers to jump out of grooves cut in reduced end of one shaft. Slotted end of hollow shaft is like a cage.



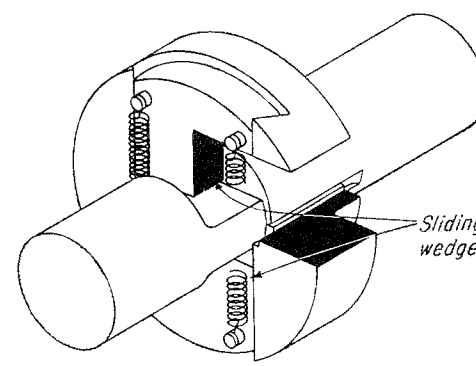
4
ARMS hold rollers in slots which are cut across disks mounted on ends of butting shafts. Springs keep rollers in slots; over-torque forces them out.



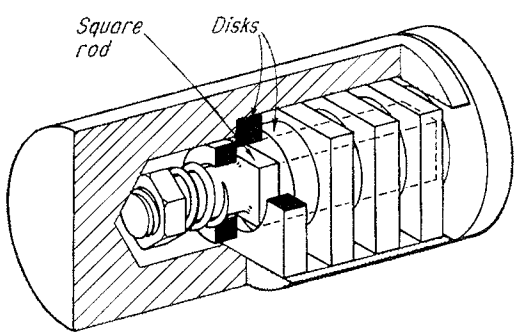
5
FLEXIBLE BELT wrapped around four pins transmits only lightest loads. Outer pins are smaller than inner pins to ensure contact.



6
SPRINGS inside drilled block grip the shaft because they distort during mounting of gear.



7
SLIDING WEDGES clamp down on flattened end of shaft; spread apart when torque gets too high. Strength of springs which hold wedges together sets torque limit.



8
FRICTION DISKS are compressed by adjustable spring. Square disks lock into square hole in left shaft; round ones lock onto square rod on right shaft.